December 9, 2025 - cascad#486

Verification Report

Title: **Algorithmic Pricing and Liquidity in Securities Markets**
Authors: **Jean-Edouard Colliard, Thierry Foucault, and Stefano Lovo**

Full reference: Colliard et al. "Algorithmic Pricing and Liquidity in Securities Markets". Accepted in *The Review of Financial Studies*. This version: November 20, 2025.

The structure and contents of this execution report provided by **cascad** for the certification are similar to those recommended by the AEA Data Editor.

1. DATA DESCRIPTION

This paper uses synthetic data from repeated simulations of the market-making game, running each set of model parameters 1,000 times to capture the randomness in client behavior, asset payoffs, and AM experimentation. This simulated data allows the authors to analyze how reinforcement learning market makers update prices, how noisy feedback leads them to misjudge the profitability of undercutting, and how their long-run pricing behavior compares to competitive benchmarks. By varying elements such as asset volatility, client valuation dispersion, adverse selection, tick size, and the number of AMs, the simulations show how different market conditions shape AM learning dynamics and pricing outcomes.

2. CODE DESCRIPTION

The verification package contains three files and is divided into 17 subfolders:

- **Figure_1/** and **Figure_6/:** Contain both one LaTeX script that must be compiled to generate the corresponding Figure.
- **Figure_2/:** Contains the Mathematica notebook *Fig_2.nb*. Running it produces the two panels of Figure 2.
- **Figure_3/** - **Figure_5/** and **Figure _7/** - **Figure_10/**: Contain MATLAB simulation scripts (*script_simulation.m*, *simulation_exp.m*, *simulation_rep.m*) and a plotting script (*script_figure_X.m*), plus **Data/** and **Figures/** subfolders.
- **Figure_11-12/:** Contains simulation scripts and plotting scripts for Figures 11 and 12 (*script_figure_11.m*, *script_figure_12.m*), plus **Data/** and **Figures/** subfolders.
- **Footnote_KS/:** Contains simulation scripts and a testing script (*script_test.m*) used to compute the KS p-values, plus a **Data**/ subfolder.
- **OnlineAppendix_Figure_OA_1/:** Contains simulation scripts and a plotting script (*script_figure_OA_1.m*), plus **Data/** and **Figures/** subfolders.

- **OnlineAppendix_Figure_OA_2_3_4/:** Contains simulation scripts for adverse selection and private values over multiple parameter combinations, and *script_figure_OA_2_3_4.m*, plus **Data/** and **Figures/** subfolders.
- **OnlineAppendix_Figure_OA_5/:** Contains simulation scripts and *script_figure_OA_5.m*, plus **Data/** and **Figures/** subfolders.
- **OnlineAppendix_Figure_OA_6-7-8/:** Contains multiple simulation regimes (baseline, exclusion, accommodation), controlled by *script_master.m*, and a plotting script (*script_figure_OA_6_7_8.m*), plus **Data/** and **Figures/** subfolders.
- **OnlineAppendix_Table_OA_1-2-3/:** Contains simulation scripts, a table-generation script (*script_table_OA_1_2_3.m*), and a MATLAB-to-LaTeX converter (*table2latex.m*), plus **Data/** and **Figures/** subfolders.

3 files:

- *Algorithmic_Pricing_and_Liquidity_in_Securities_Markets.pdf*
- *List_Figures.xlsx*
- *README_CODE_ONLY.txt*

3. VERIFICATION STEPS

The code was delivered by email on December 3rd, 2025, and run as per readme, using Matlab R2023b and Wolfram Mathematica 14.2. To accelerate the process, we used four machines, all running Windows 11 Enterprise:

- **Machine 1:** Intel(R) Xeon(R) w9-3575X CPU (44 cores) with 256 GB RAM.
- **Machine 2:** Intel Xeon Silver 4216 2.10GHz CPU (32 cores) with 256 GB RAM.
- **Machine 3:** Intel(R) Xeon(R) Silver 4216 2.10GHz CPU (32 cores) with 512 GB RAM.
- **Machine 4:** Intel(R) Xeon(R) w9-3475X CPU 2.21 GHz (36 cores) with 256 GB RAM.

We encountered no issues during the verification.

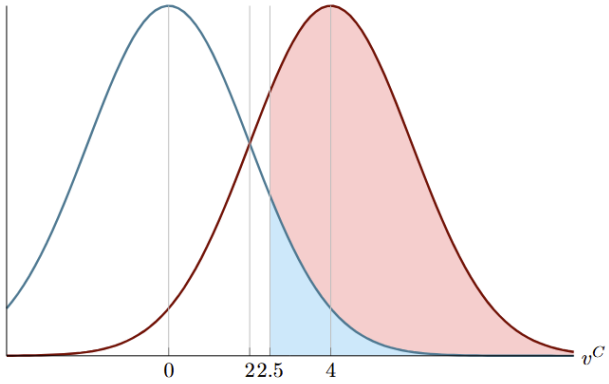
4. FINDINGS

Figures:
We reproduced Figures 1-3, 6-7, 10-12, and OA.5-OA.6 with accuracy.
We reproduced Figures 4-5, 8-9, and OA.2-OA.4, and OA.7 with slight discrepancies.

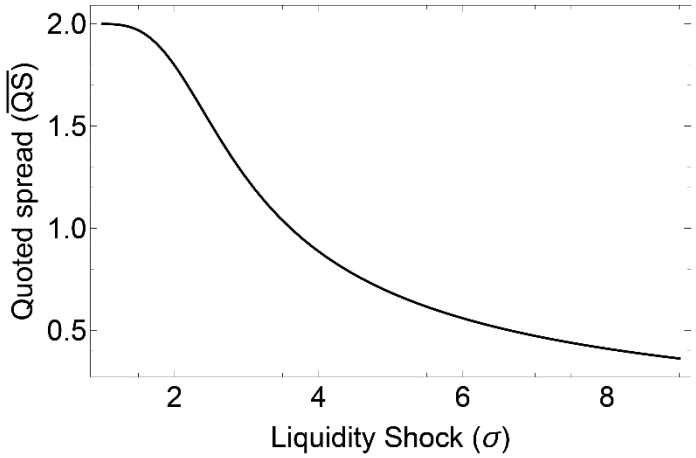
Tables:
We reproduced Table OA.1-OA.3 with slight discrepancies. Discrepancies higher than rounding errors in these Tables are highlighted in orange.

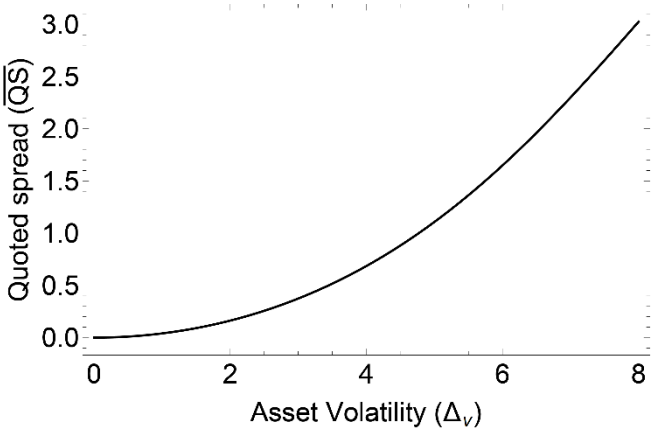
Note that, due to the use of parallelisation and Monte-Carlo methods in the code, regenerating the results from scratch with perfect accuracy is impossible even when setting the seed value (for more details, please refer to the authors' readme file).

4.1. FIGURE 1: DISTRIBUTION OF THE CLIENT'S VALUATION



4.2. FIGURE 2: GLOSTEN-MILGROM BENCHMARK.

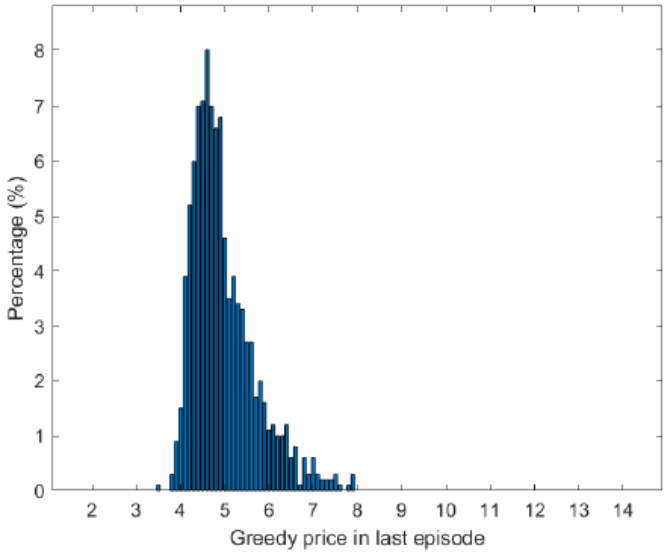




4.3. FIGURE 3: GREEDY PRICE OF AM1 IN THE ADVERSE SELECTION CASE.

PANEL A:

ORIGINAL:



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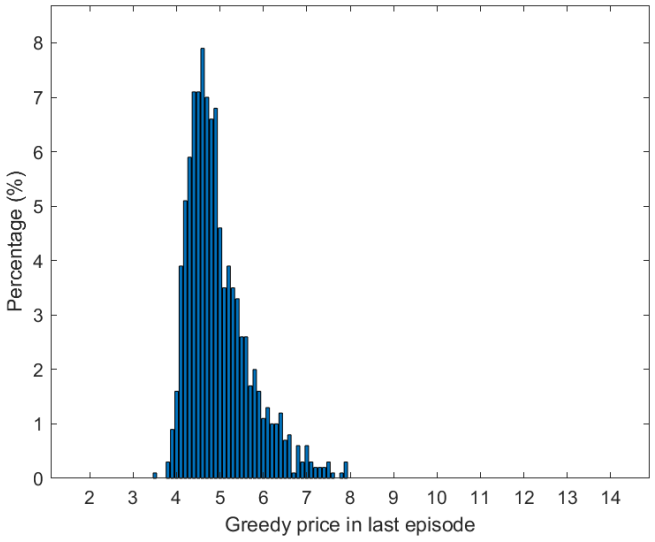
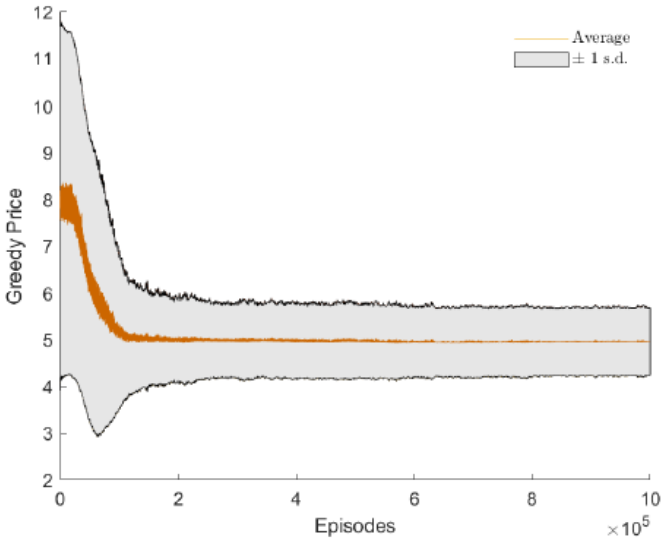


PANEL B:

ORIGINAL:



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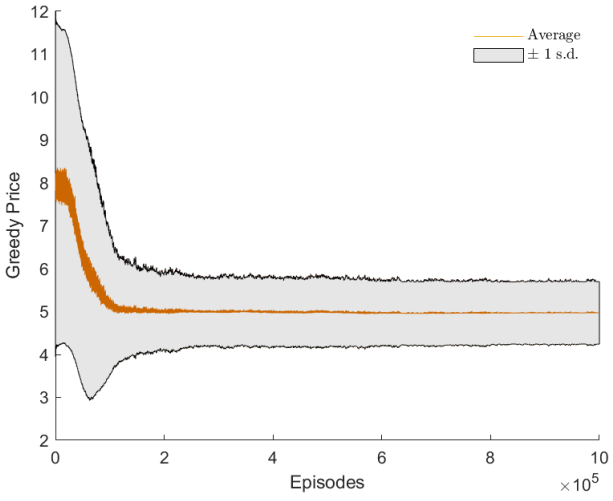
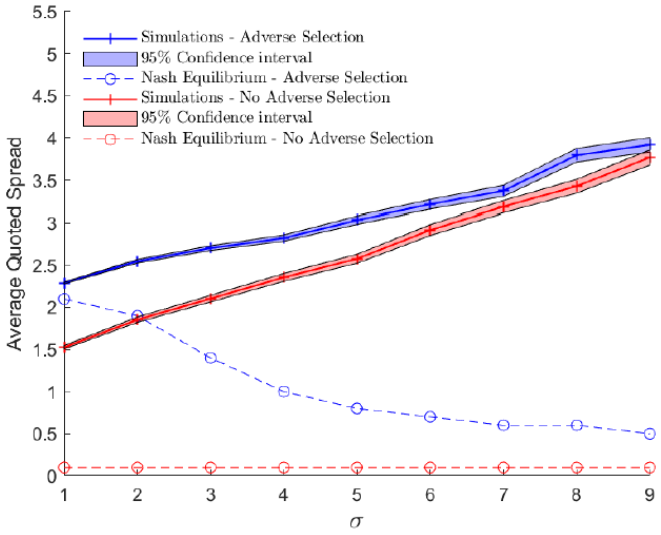
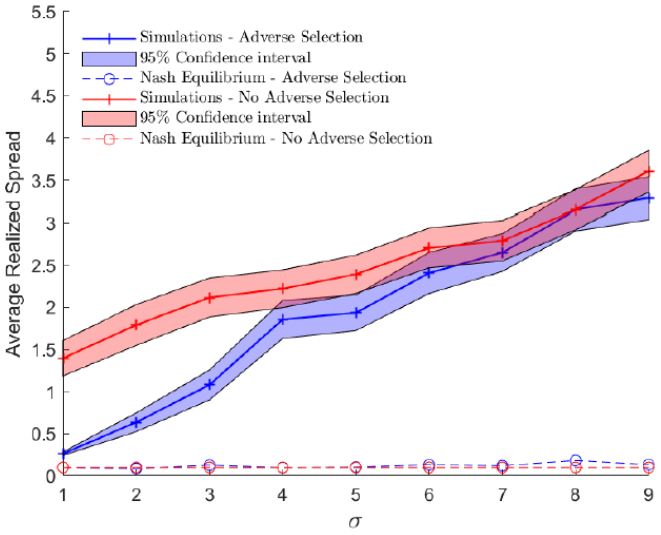


4.4. FIGURE 4: AVERAGE QUOTED SPREAD QS AND AVERAGE REALIZED SPREAD RS IN THE ADVERSE SELECTION CASE AND THE NO ADVERSE SELECTION CASE, FOR DIFFERENT VALUES OF THE DISPERSION OF CLIENTS' LIQUIDITY SHOCKS σ

ORIGINAL:

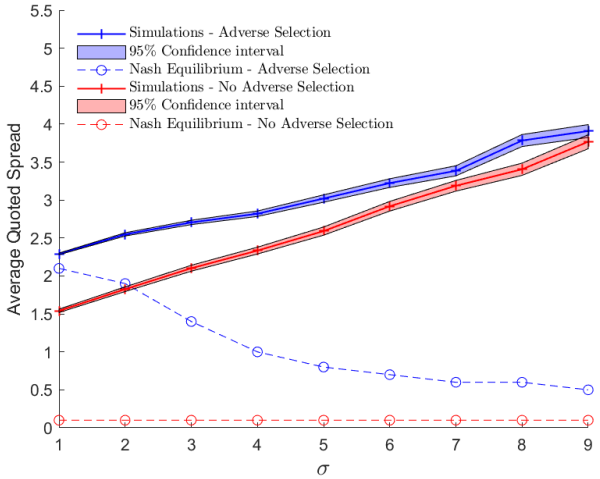


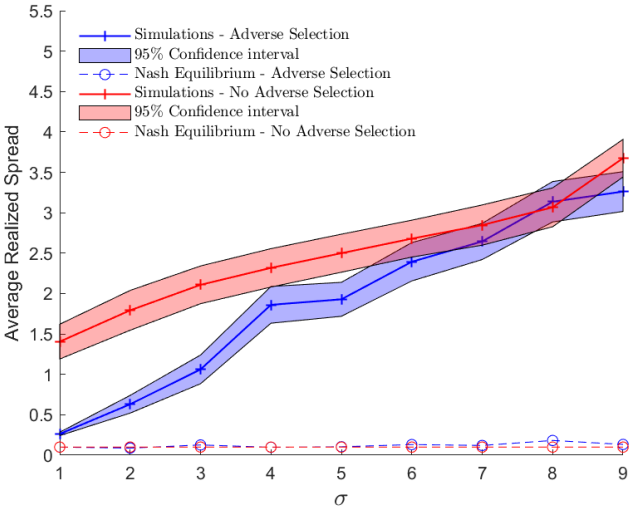
Panel A: Average Quoted Spread $\overline{QS}$



Panel B: Average Realized Spread $\overline{RS}$

REPRODUCED:

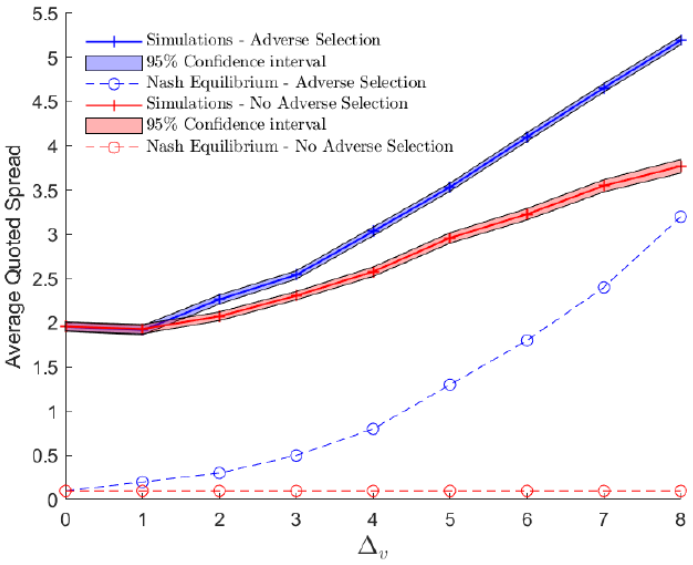




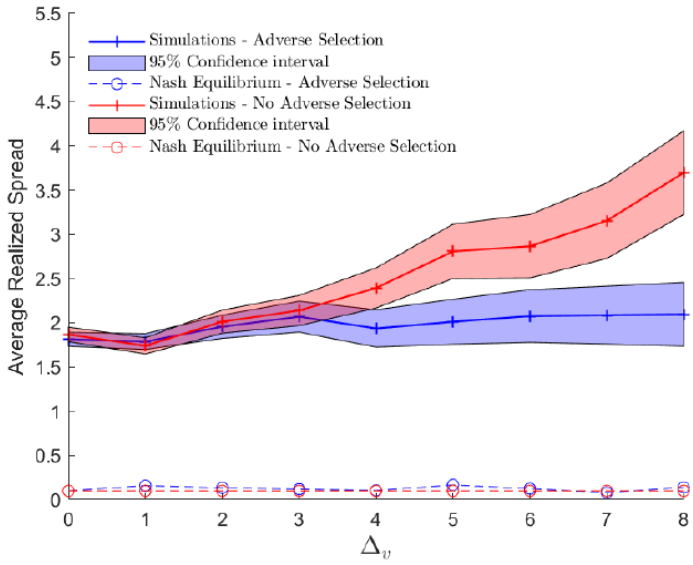
The dotted lines are identical in the original Figures and in ours. On the other hand, the continuous lines and the shaded areas have similar shape but are slightly different.

4.5. FIGURE 5: AVERAGE QUOTED SPREAD QS AND AVERAGE REALIZED SPREAD RS IN THE ADVERSE SELECTION CASE AND THE NO ADVERSE SELECTION CASE, FOR DIFFERENT VALUES OF THE ASSET VOLATILITY ΔV.

ORIGINAL:

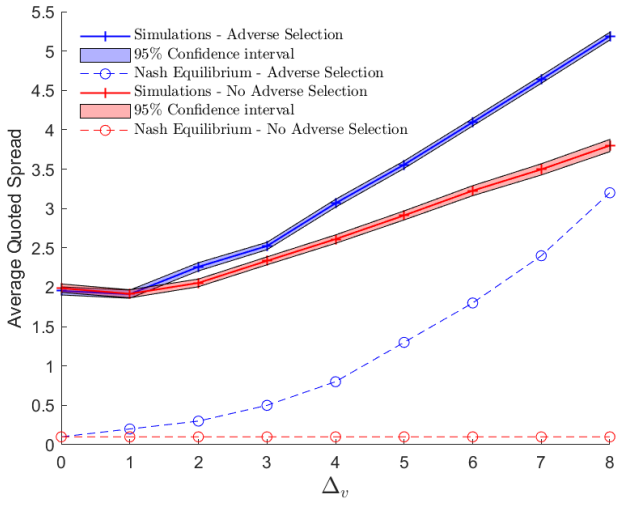


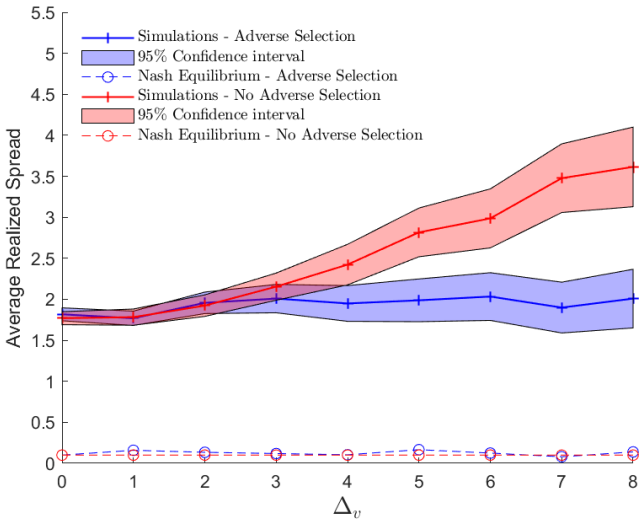
Panel A: Average Quoted Spread $\overline{QS}$



Panel B: Average Realized Spread $\overline{RS}$

REPRODUCED:

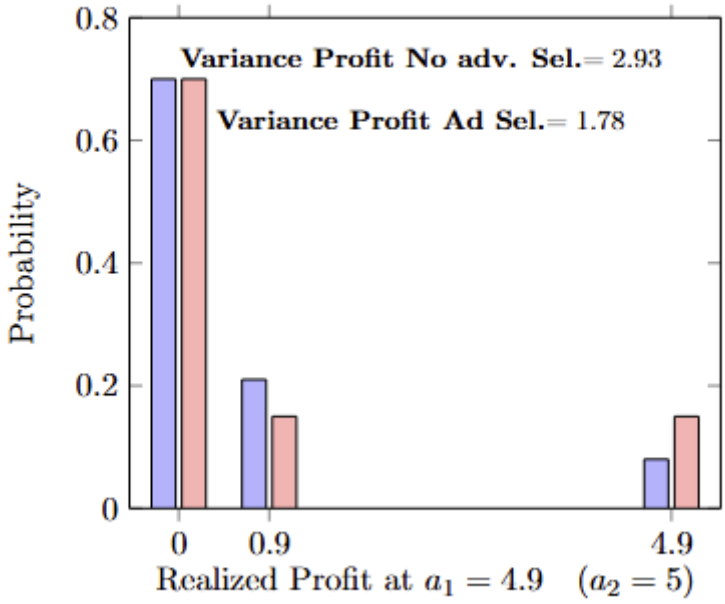




The dotted lines are identical in the original Figures and in ours. On the other hand, the continuous lines and the shaded areas have similar shape but are slightly different.

4.6. FIGURE 6: DISTRIBUTION OF PROFITS WITH AND WITHOUT ADVERSE SELECTION.

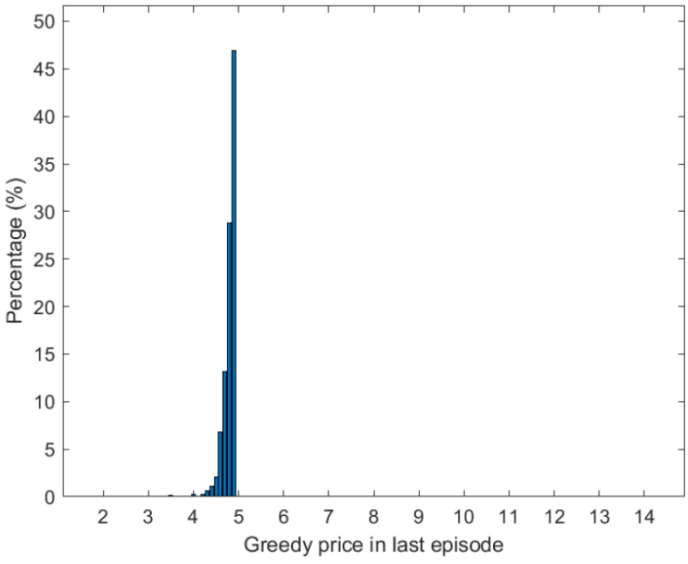
REPRODUCED:



4.7. FIGURE 7: GREEDY PRICE OF AM1 WHEN AM2 PLAYS A CONSTANT PRICE IN THE ADVERSE SELECTION CASE.

PANEL A:

ORIGINAL:



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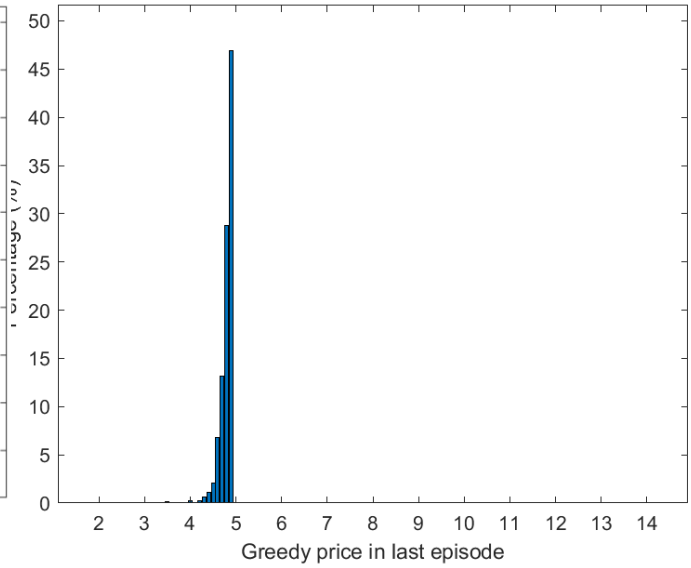

PANEL B:

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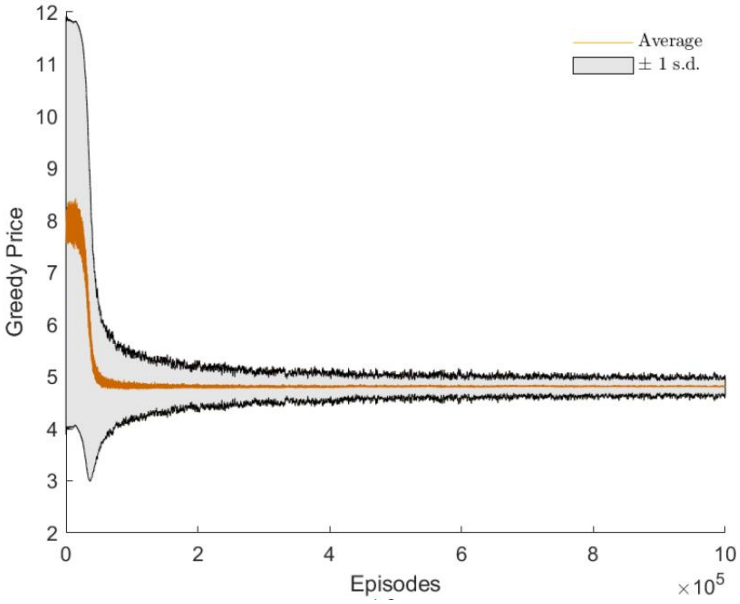

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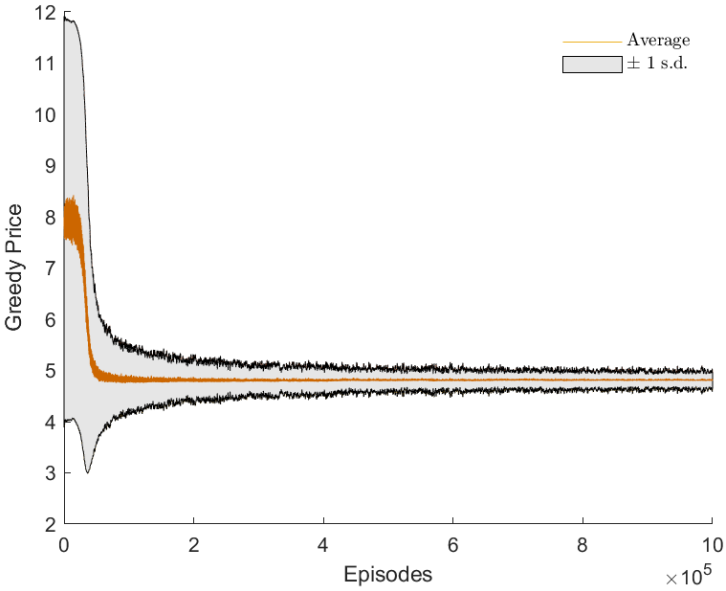
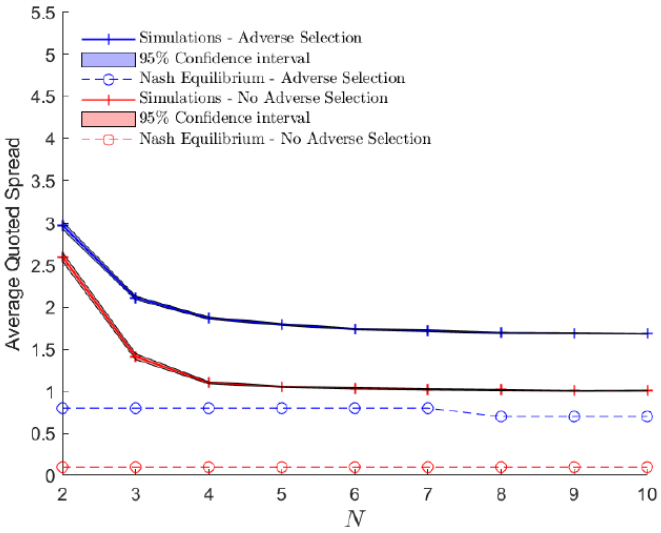
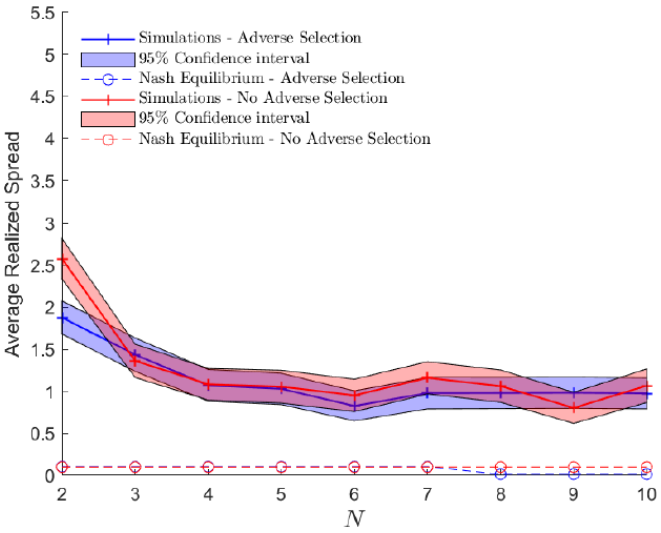


4.8. FIGURE 8: AVERAGE QUOTED SPREAD QS AND AVERAGE REALIZED SPREAD RS IN THE ADVERSE SELECTION CASE AND THE NO ADVERSE SELECTION CASE, FOR DIFFERENT VALUES OF THE NUMBER N OF AMS.

ORIGINAL:

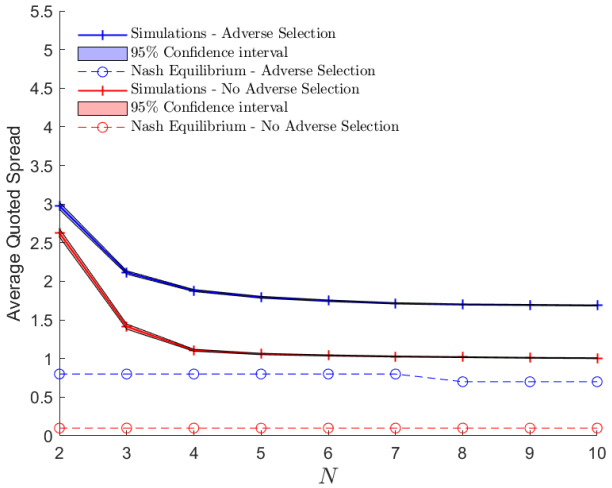


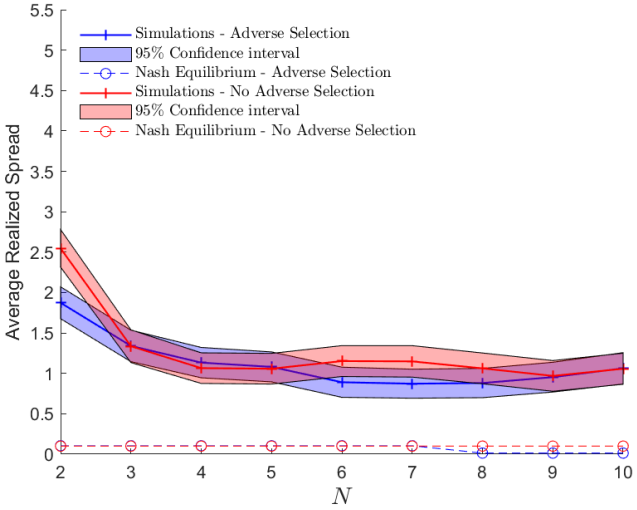
Panel A: Average Quoted Spread $\overline{QS}$



Panel B: Average Realized Spread $\overline{RS}$

REPRODUCED:

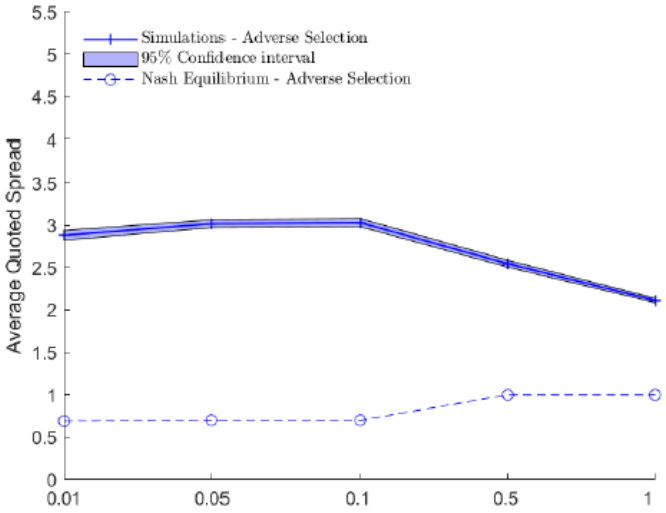




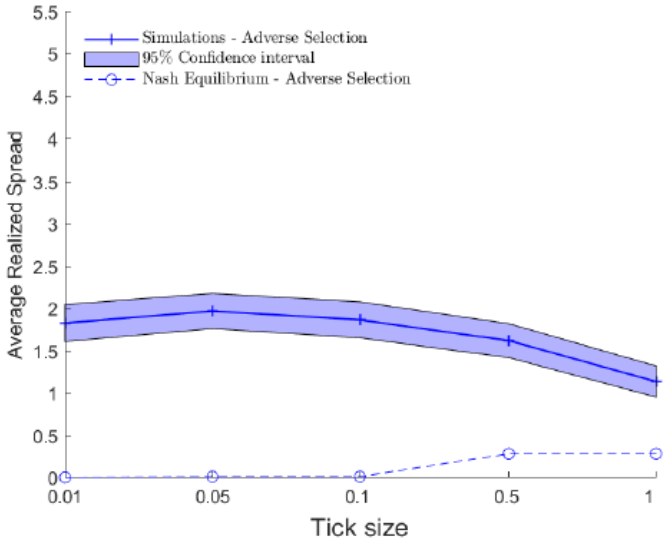
Panel A is reproduced with accuracy.
As for Panel B, the dotted lines are identical in the original Figures and in ours, but the continuous lines and the shaded areas are slightly different.

4.10. FIGURE 9: AVERAGE QUOTED SPREAD QS AND AVERAGE REALIZED SPREAD RS IN THE ADVERSE SELECTION CASE, FOR DIFFERENT VALUES OF THE TICK SIZE.

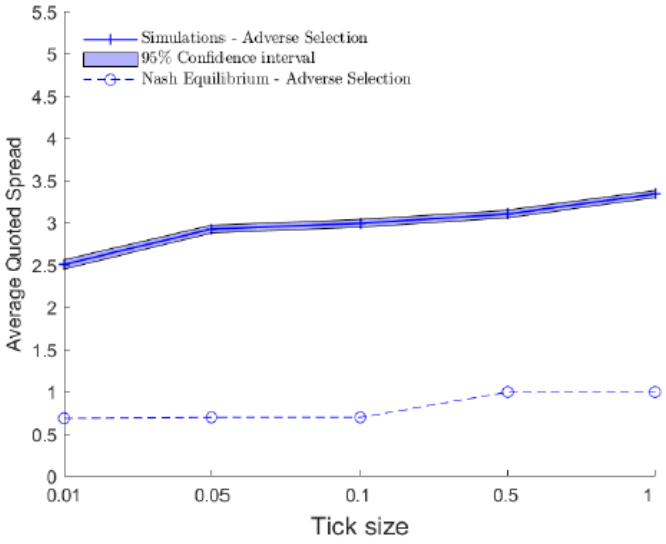
ORIGINAL:



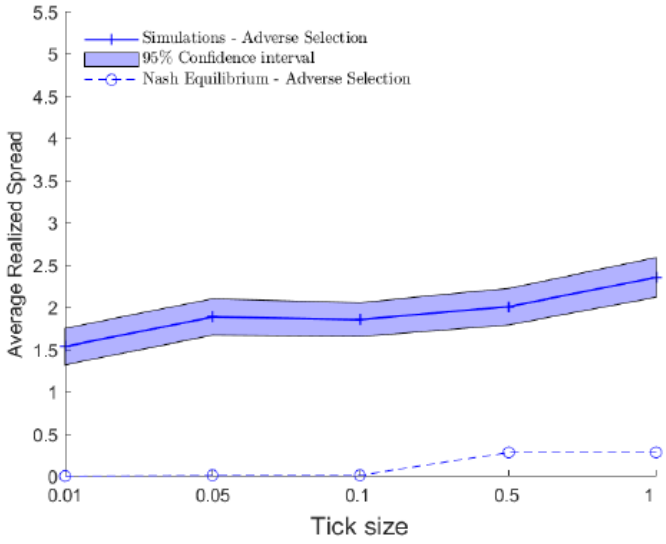
Panel A: Average Quoted Spread $\overline{QS}$



Panel B: Average Realized Spread $\overline{RS}$

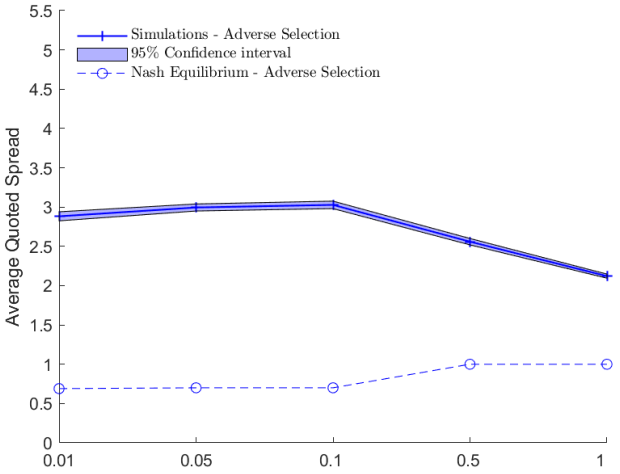


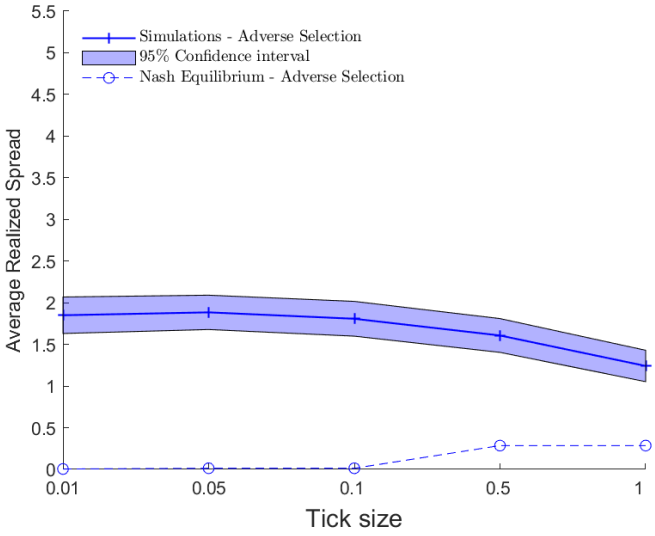
Panel C: Average Quoted Spread $\overline{QS}$, adjusted β

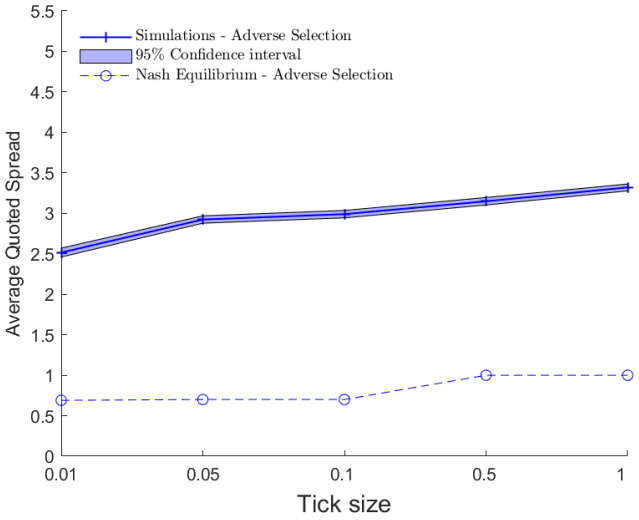


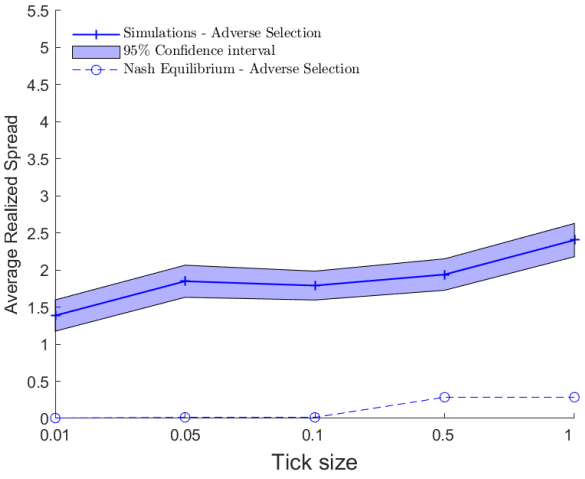
Panel D: Average Realized Spread $\overline{RS}$, adjusted β

REPRODUCED:







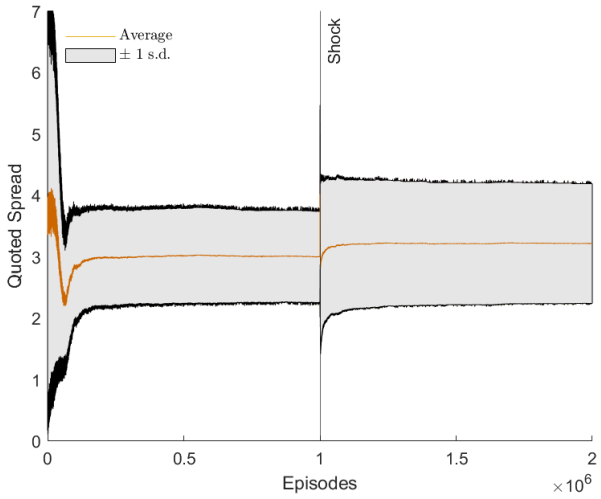


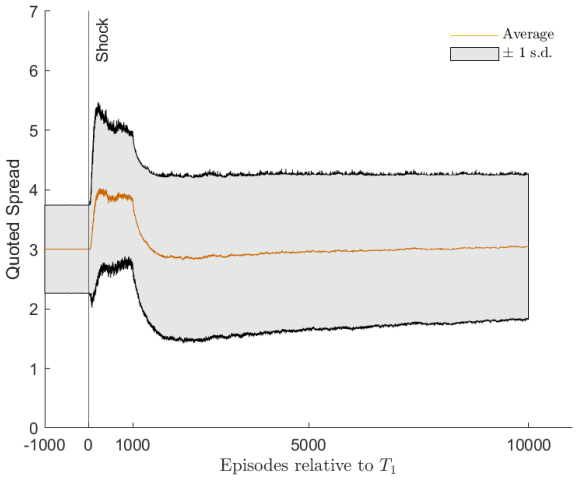
The dotted lines are identical.

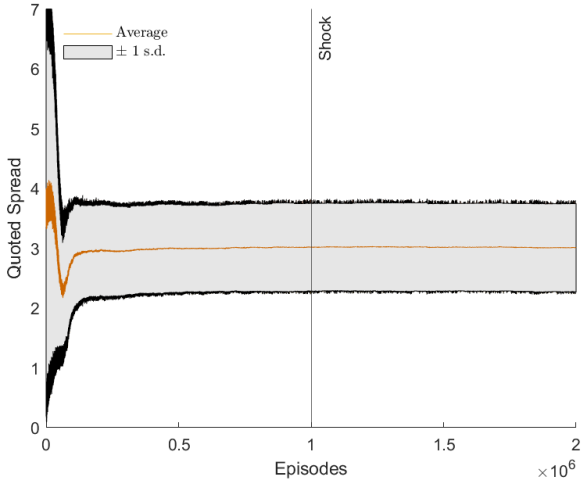
The continuous lines and the shaded areas that surround them, they are almost identical except for a few discrepancies.

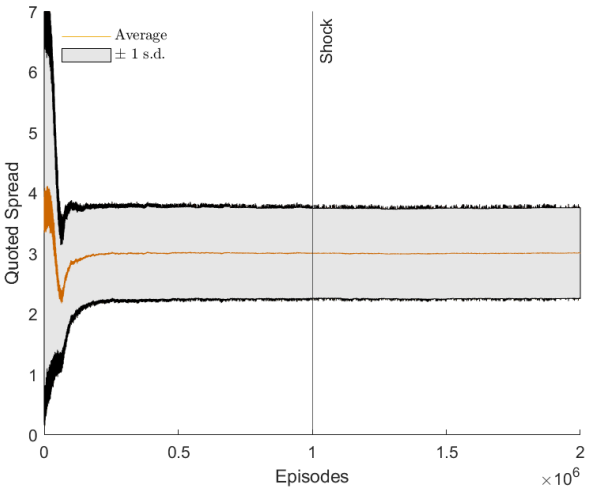
4.11. FIGURE 10: DYNAMICS OF QUOTED SPREADS AFTER A SHOCK ON ADVERSE SELECTION.

REPRODUCED:



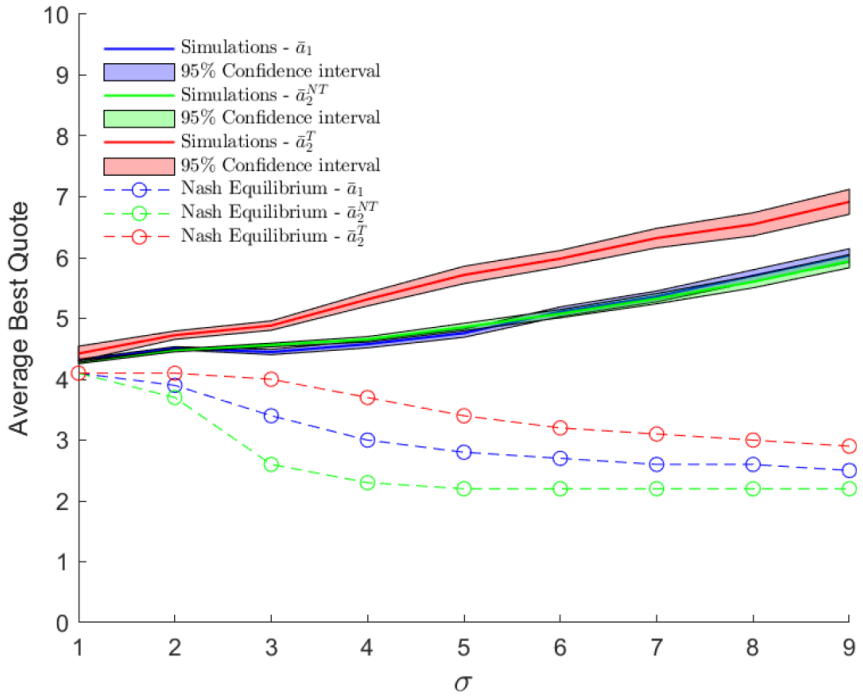






4.12. FIGURE 11: AVERAGE FIRST-PERIOD PRICE A1 AND SECOND-PERIOD PRICE AFTER A TRADE A2T AND AFTER NO TRADE A2NT, FOR DIFFERENT VALUES OF THE DISPERSION OF CLIENTS' LIQUIDITY SHOCKS Σ.

ORIGINAL:



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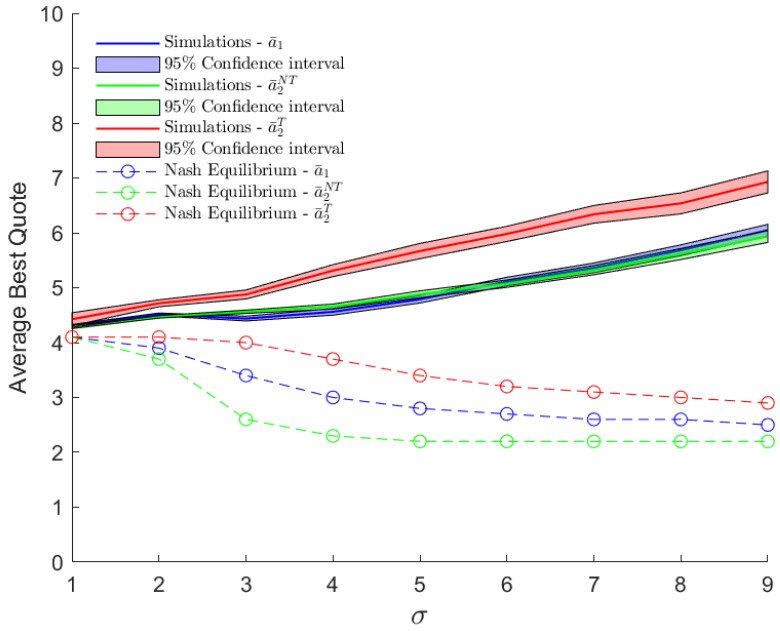
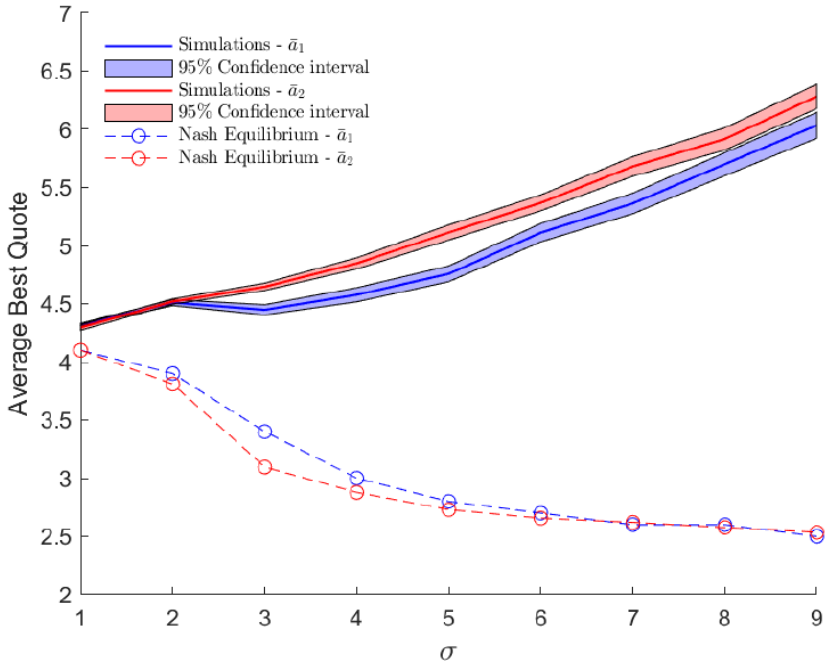


4.13. FIGURE 12: AVERAGE FIRST-PERIOD PRICE A1 AND AVERAGE SECOND-PERIOD PRICE A2, FOR DIFFERENT VALUES OF THE DISPERSION OF CLIENTS' LIQUIDITY SHOCKS σ.

ORIGINAL:



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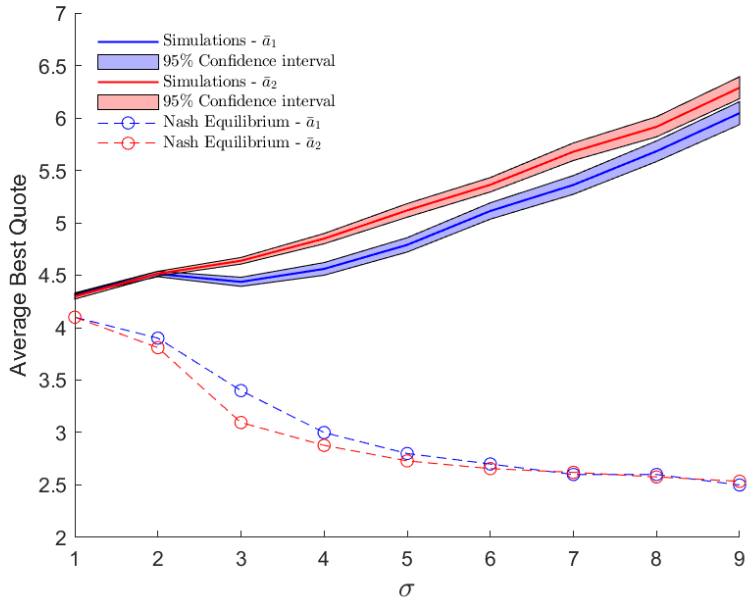

4.14. FIGURE OA.1: GREEDY PRICE OF AM1 WHEN AM1 AND AM2 KEEP EXPERIMENTING IN THE LONG-RUN.

PANEL A:

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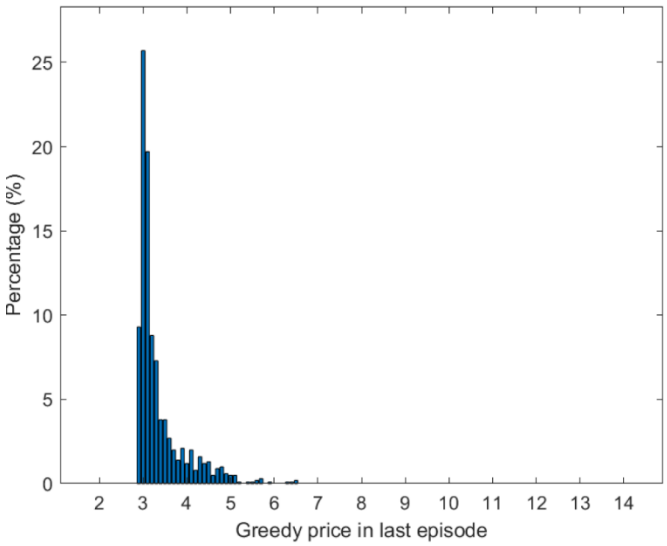

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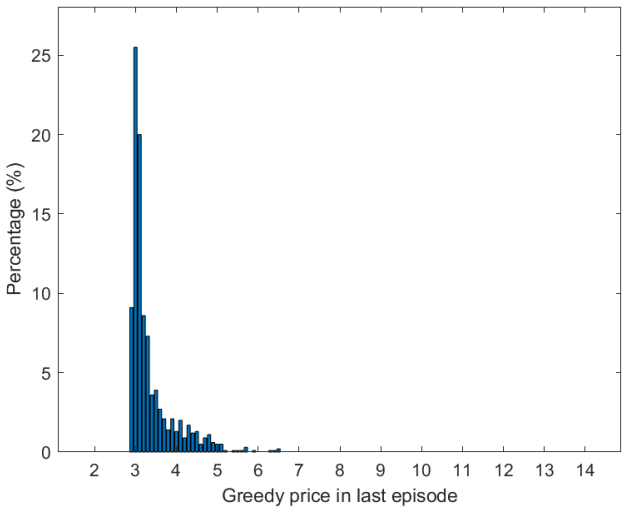
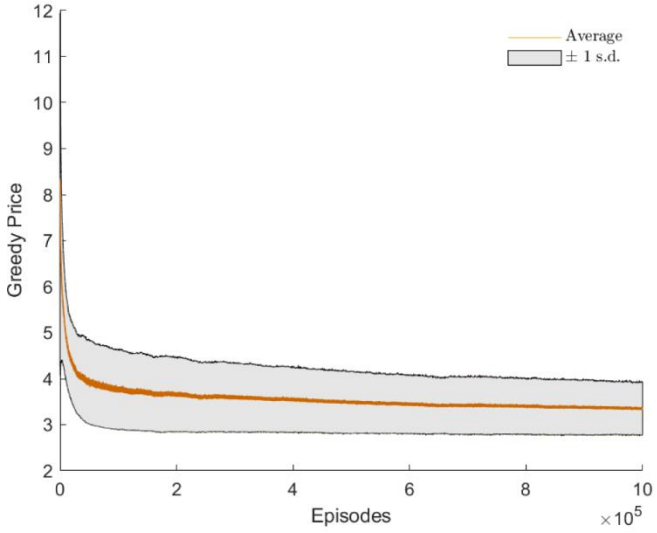


PANEL B:

ORIGINAL:



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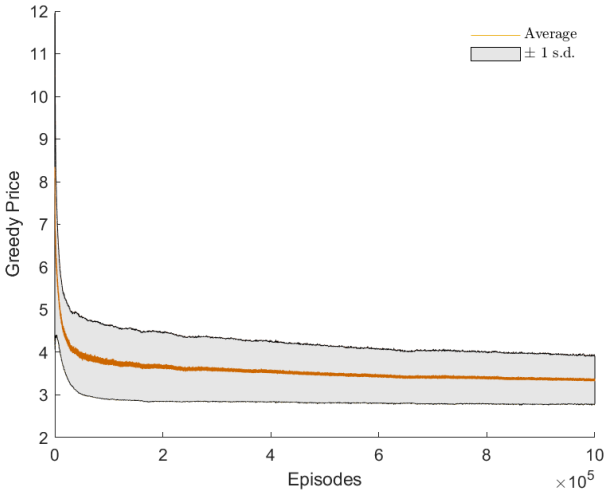


4.15. TABLE OA.1: FINAL PRICE FOR DIFFERENT HYPERPARAMETERS.

ORIGINAL:

		AM_2								
		(α_l, β_l)	(α_l, β_m)	(α_l, β_h)	(α_m, β_l)	(α_m, β_m)	(α_m, β_h)	(α_h, β_l)	(α_h, β_m)	(α_h, β_h)
AM_1	(α_l, β_l)	5.37	5.42	5.39	4.26	4.34	4.34	3.92	3.99	3.98
	(α_l, β_m)	5.42	5.73	5.71	4.19	4.81	4.8	3.88	4.01	4
	(α_l, β_h)	5.39	5.71	5.69	4.19	4.83	4.84	3.88	3.99	4
	(α_m, β_l)	4.26	4.19	4.19	4.11	4.1	4.14	3.98	4.08	4.07
	(α_m, β_m)	4.34	4.81	4.83	4.1	5.03	5.04	3.87	4.4	4.41
	(α_m, β_h)	4.34	4.8	4.84	4.14	5.04	5.07	3.86	4.43	4.43
	(α_h, β_l)	3.92	3.88	3.88	3.98	3.87	3.86	4	4.01	4.01
	(α_h, β_m)	3.99	4.01	3.99	4.08	4.4	4.43	4.01	4.15	4.18
	(α_h, β_h)	3.98	4	4	4.07	4.41	4.43	4.01	4.18	4.34

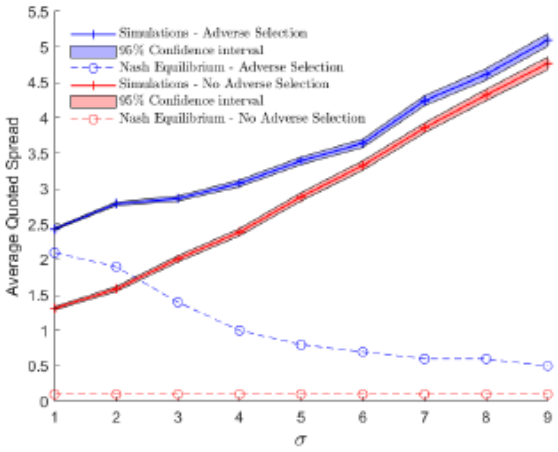
REPRODUCED:

$MatPrices_1 1$	$MatPrices_1 2$	$MatPrices_1 3$	$MatPrices_1 4$	$MatPrices_1 5$	$MatPrices_1 6$	$MatPrices_1 7$	$MatPrices_1 8$	$MatPrices_1 9$
5.37	5.42	5.39	4.25	4.3	4.34	3.92	3.97	3.98
5.42	5.73	5.71	4.19	4.79	4.78	3.88	4.01	4
5.39	5.71	5.69	4.21	4.8	4.84	3.88	4	4
4.25	4.19	4.21	4.1	4.1	4.13	3.98	4.08	4.09
4.3	4.79	4.8	4.1	5	5.02	3.87	4.42	4.42
4.34	4.78	4.84	4.13	5.02	5.07	3.85	4.43	4.41
3.92	3.88	3.88	3.98	3.87	3.85	3.99	4.01	4.01
3.97	4.01	4	4.08	4.42	4.43	4.01	4.16	4.19
3.98	4	4	4.09	4.42	4.41	4.01	4.19	4.35

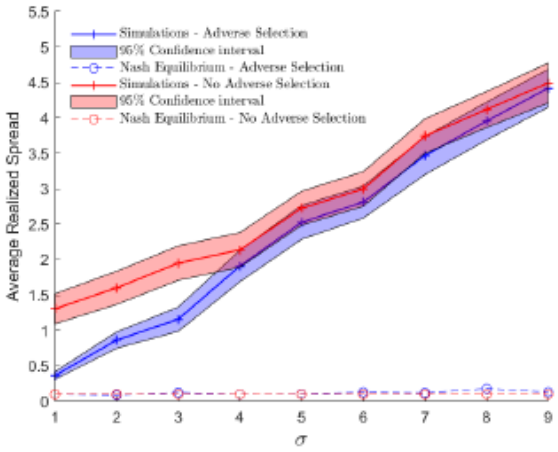
Overall, the values we found are very close to those displayed in the paper. The biggest difference we found id 0.04 (first row, fifth column; and fifth row, first column)

4.16. FIGURE OA.2: AVERAGE QUOTED SPREAD AND REALIZED SPREAD IN THE LAST EPISODE, FOR DIFFERENT VALUES OF σ .

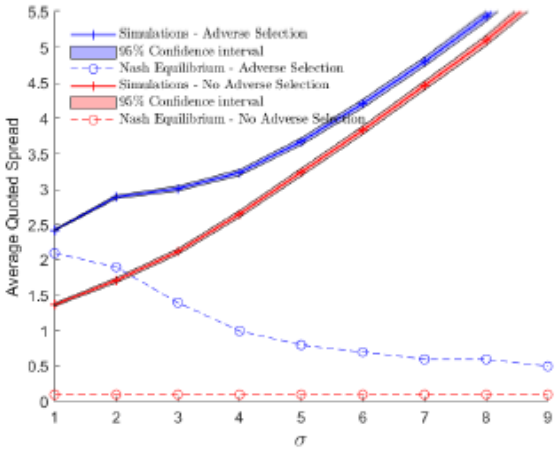
ORIGINAL:



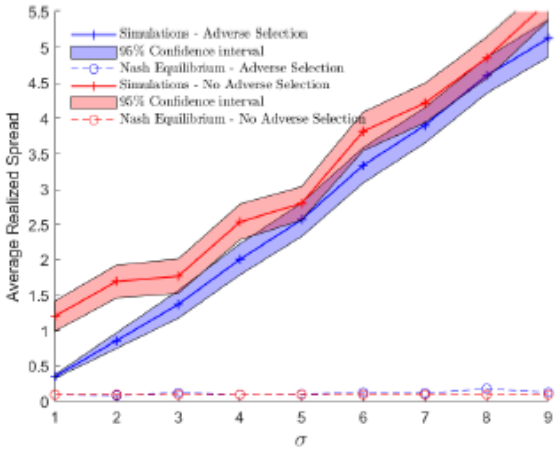
Quoted spread, $\alpha = 0.001, \beta = 5 \times 10^{-6}$



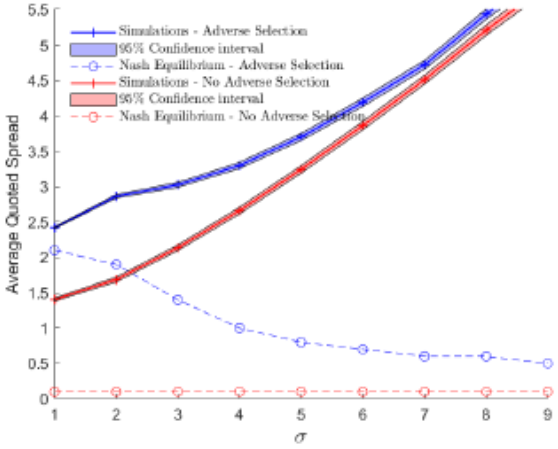
Realized spread, $\alpha = 0.001, \beta = 5 \times 10^{-6}$



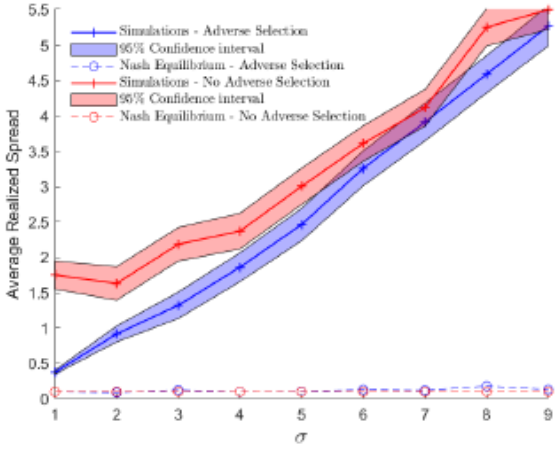
Quoted spread, $\alpha = 0.001, \beta = 8 \times 10^{-5}$



Realized spread, $\alpha = 0.001, \beta = 8 \times 10^{-5}$

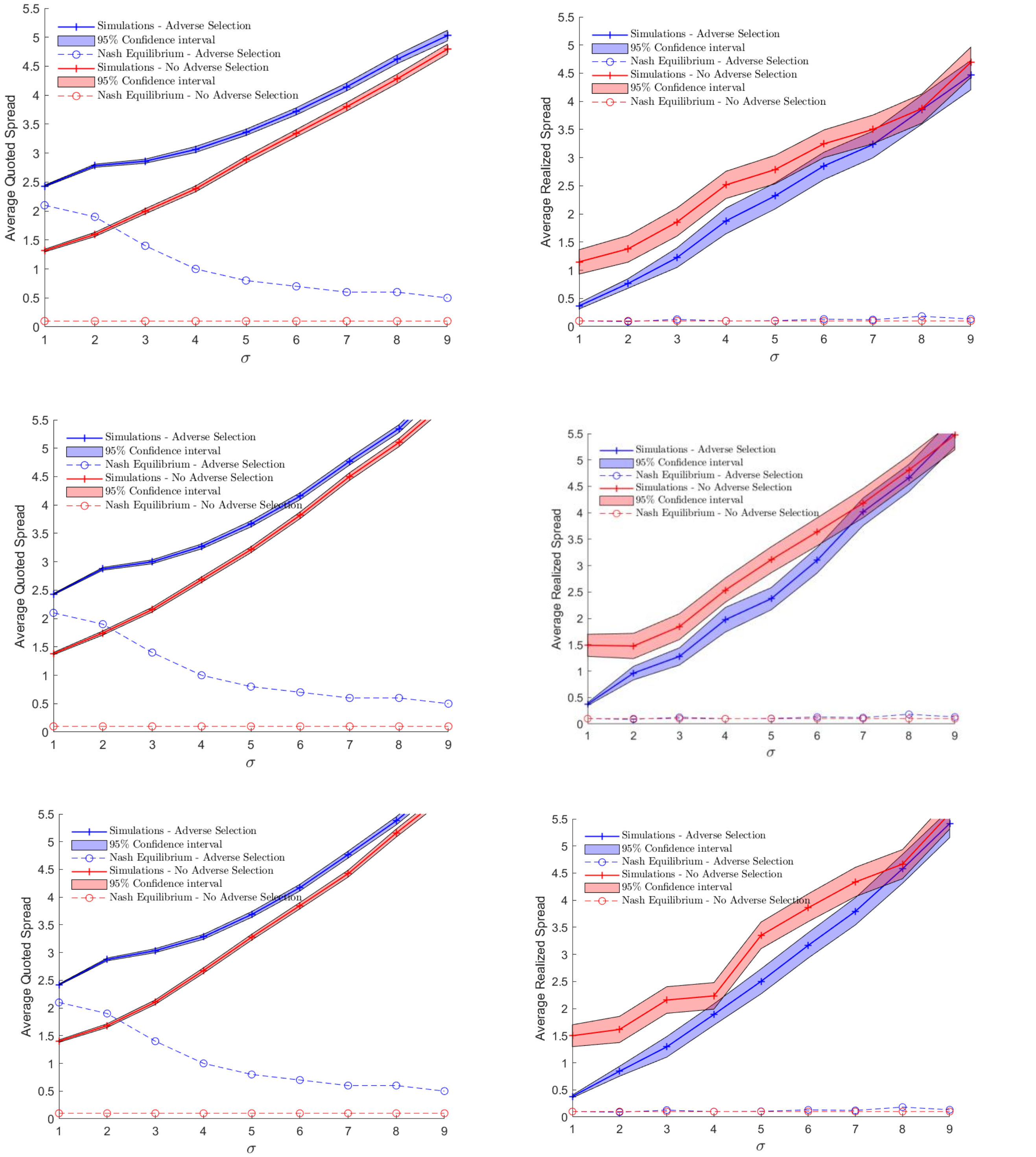


Quoted spread, $\alpha = 0.001, \beta = 3.2 \times 10^{-4}$



Realized spread, $\alpha = 0.001, \beta == 3.2 \times 10^{-4}$

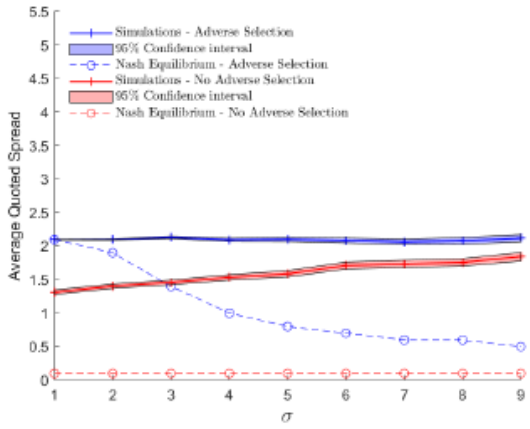
REPRODUCED:



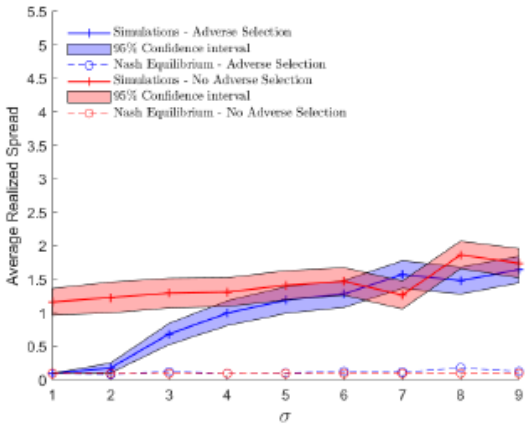
The dotted lines are identical.
The continuous lines and the shaded areas that surround them have similar shape but are slightly different.

4.17. FIGURE OA.3: AVERAGE QUOTED SPREAD AND REALIZED SPREAD IN THE LAST EPISODE, FOR DIFFERENT VALUES OF σ.

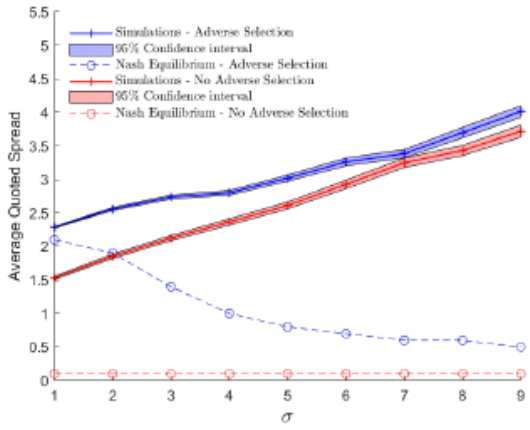
ORIGINAL:



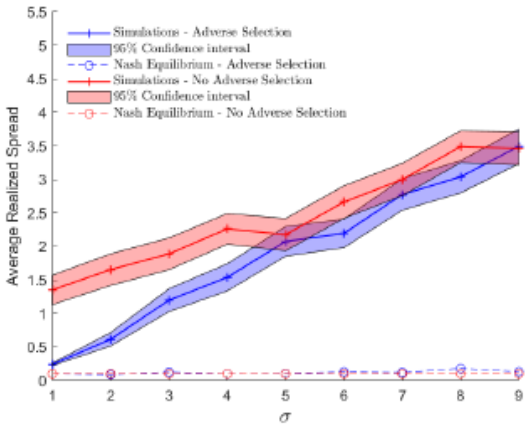
Quoted spread, $\alpha = 0.01, \beta = 5 \times 10^{-6}$



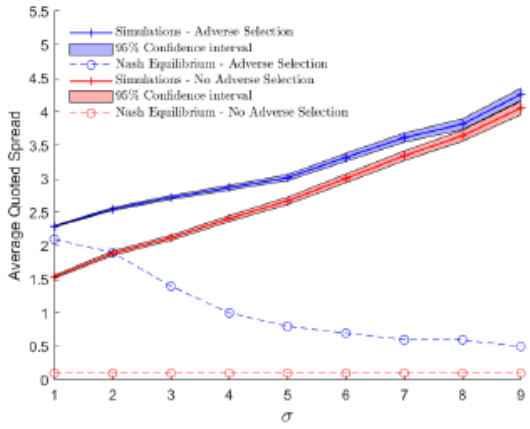
Realized spread, $\alpha = 0.01, \beta = 5 \times 10^{-6}$



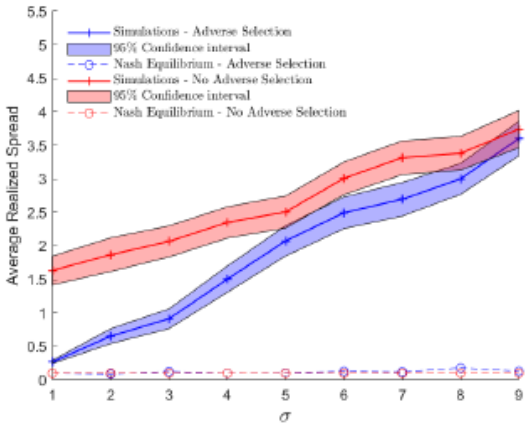
Quoted spread, $\alpha = 0.01, \beta = 8 \times 10^{-5}$



Realized spread, $\alpha = 0.01, \beta = 8 \times 10^{-5}$



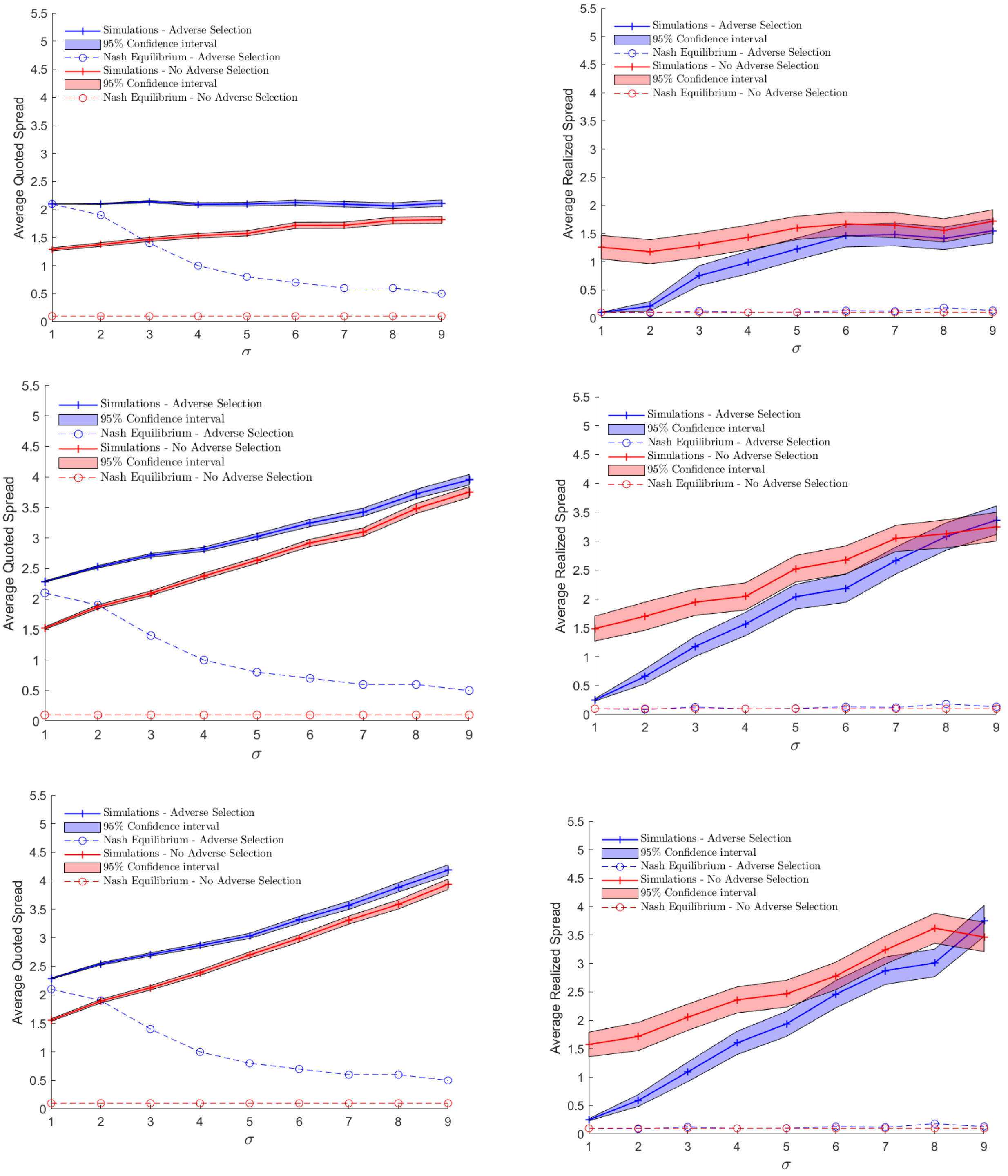
Quoted spread, $\alpha = 0.01, \beta = 3.2 \times 10^{-4}$



Realized spread, $\alpha = 0.01, \beta == 3.2 \times 10^{-4}$

69

REPRODUCED:

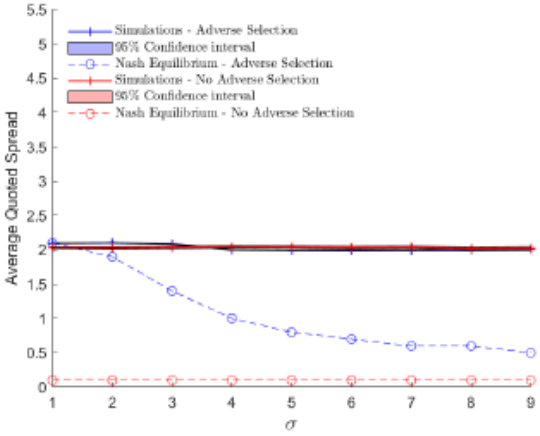


The dotted lines are identical.

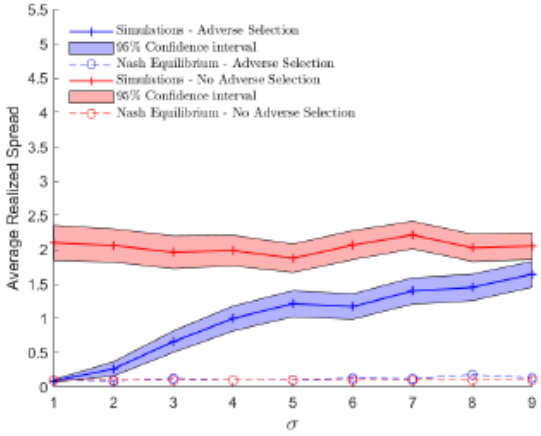
The continuous lines and the shaded areas that surround them have similar shape but are slightly different.

4.18. FIGURE OA.4: AVERAGE QUOTED SPREAD AND REALIZED SPREAD IN THE LAST EPISODE, FOR DIFFERENT VALUES OF σ .

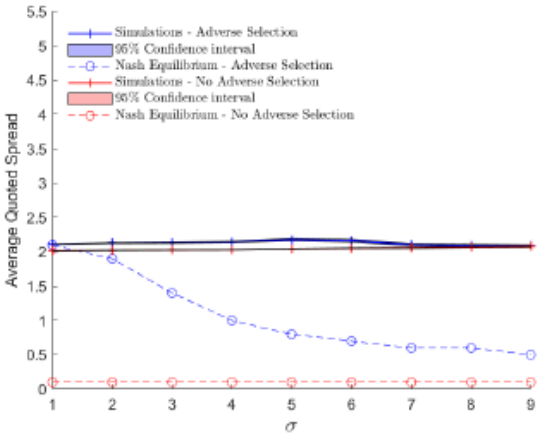
ORIGINAL:



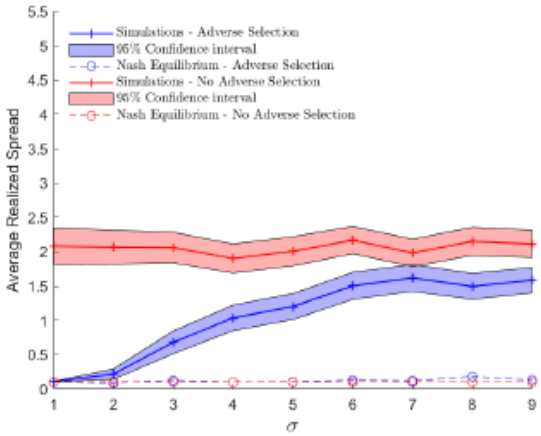
Quoted spread, $\alpha = 0.1, \beta = 5 \times 10^{-6}$



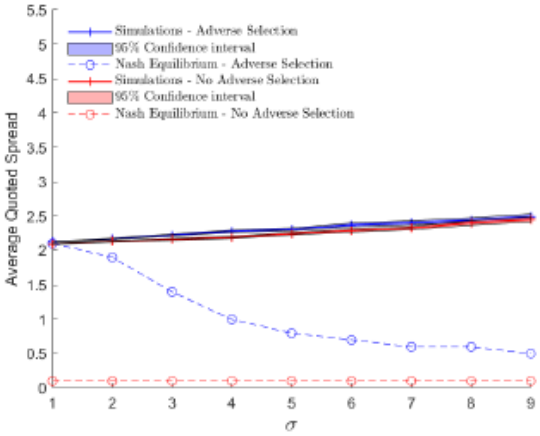
Realized spread, $\alpha = 0.1, \beta = 5 \times 10^{-6}$



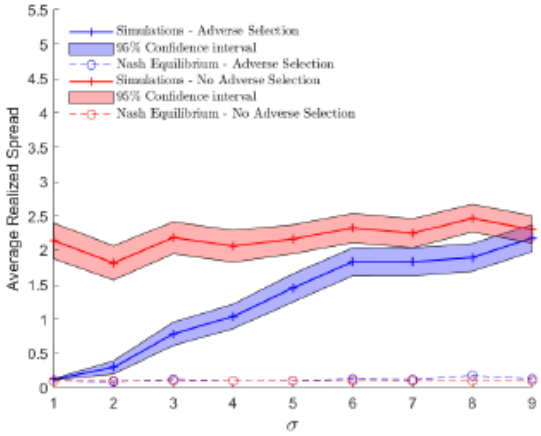
Quoted spread, $\alpha = 0.1, \beta = 8 \times 10^{-5}$



Realized spread, $\alpha = 0.1, \beta = 8 \times 10^{-5}$

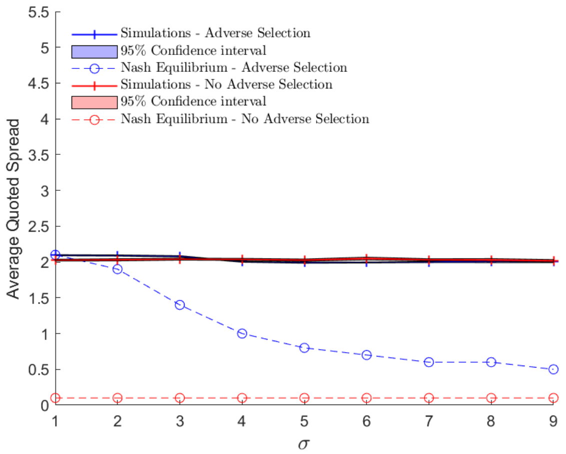


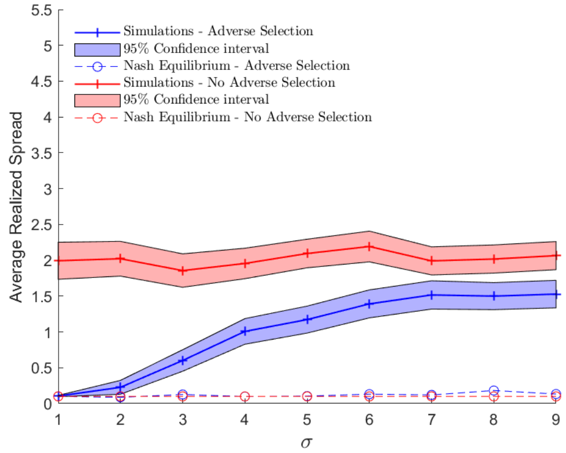
Quoted spread, $\alpha = 0.1, \beta = 3.2 \times 10^{-4}$

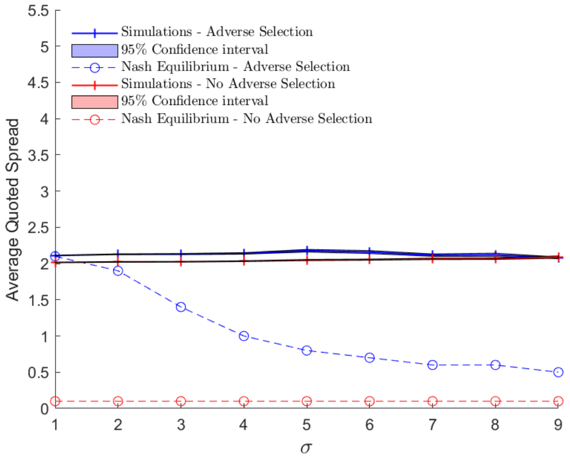


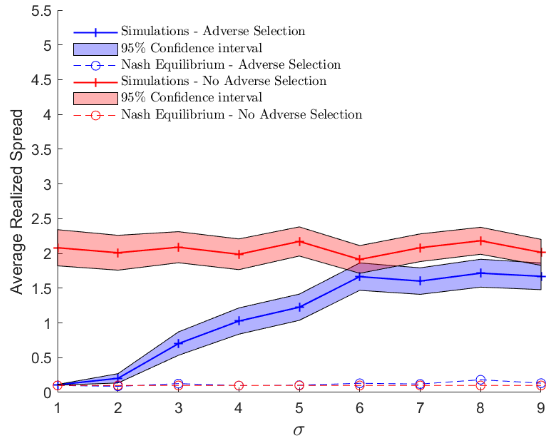
Realized spread, $\alpha = 0.1, \beta == 3.2 \times 10^{-4}$

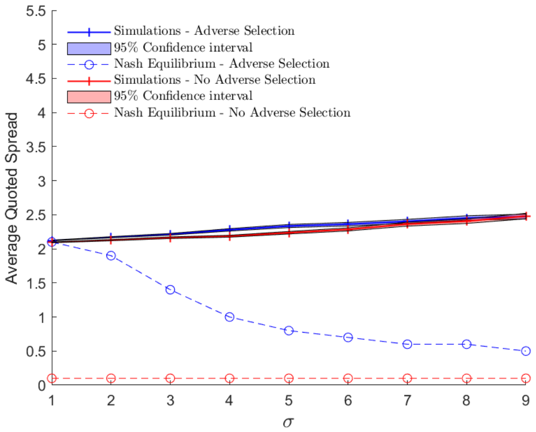
REPRODUCED:

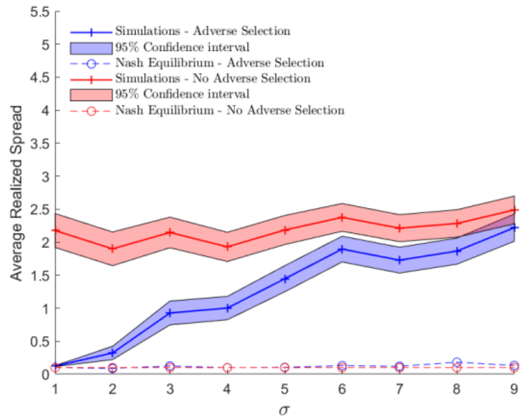












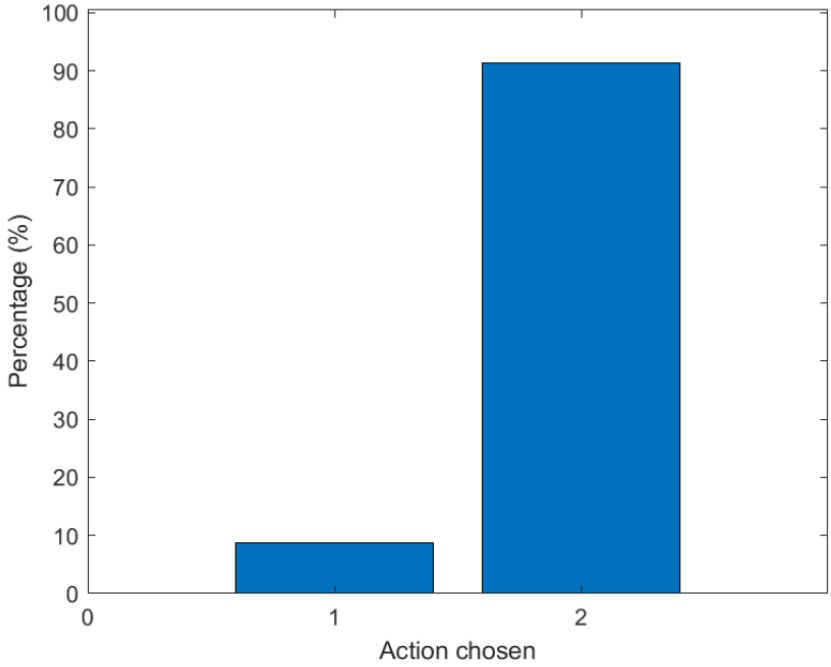
The dotted lines are identical.

The continuous lines and the shaded areas that surround them have similar shape but are slightly different.

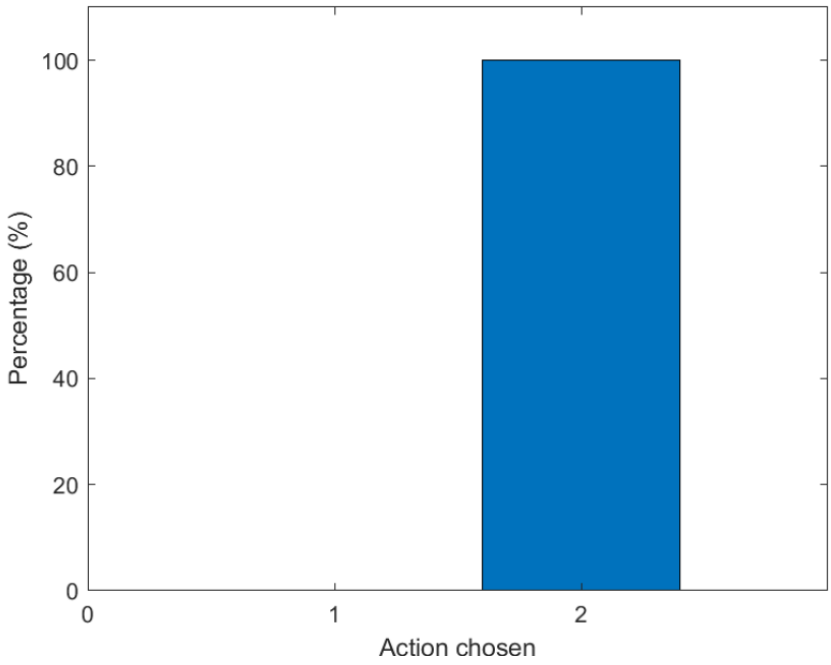
4.19. FIGURE OA.5: PERCENTAGE OF EXPERIMENTS ENDING WITH ACTIONS A1 AND A2, FIXED STOPPING TIME AND RANDOM STOPPING TIME PROCEDURES.

ORIGINAL:

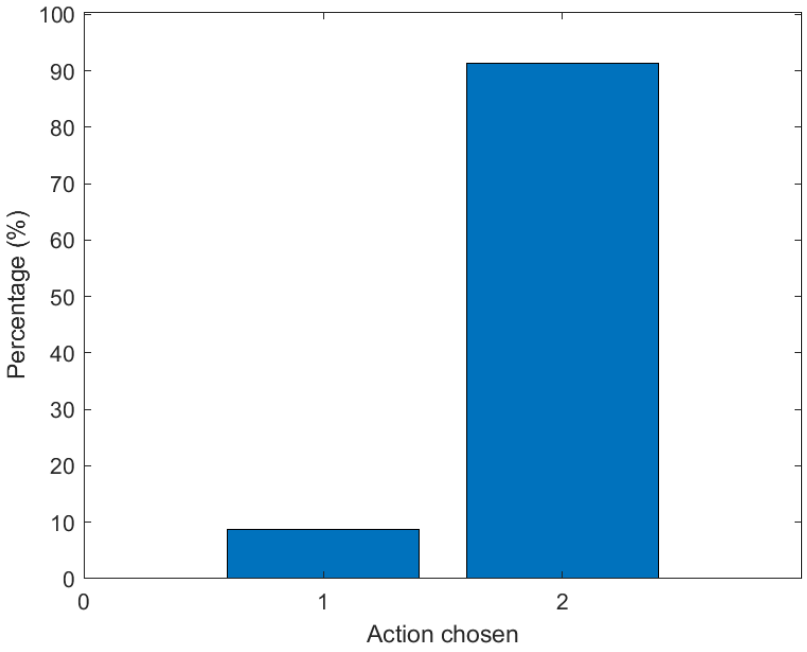
Panel A: Fixed stopping time procedure.

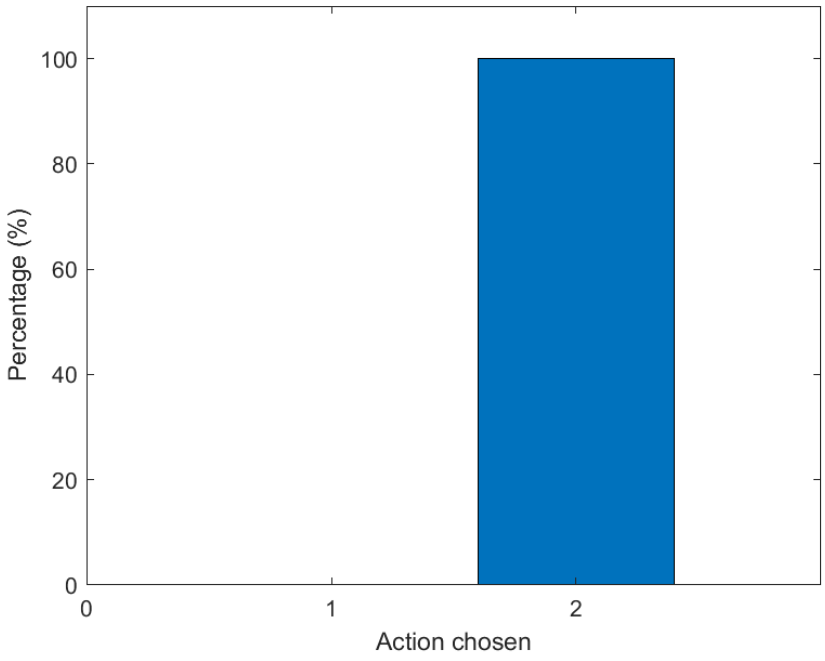


Panel B: Random stopping time procedure.



REPRODUCED:


100
90
80
70
60
50
40
30
20
10
0
Percentage (%)
0
1
2
Action chosen


100
80
60
40
20
0
Percentage (%)
0
1
2
Action chosen

4.20. TABLE OA.2: DEVIATIONS FROM (α M, β M).

ORIGINAL:

θ'	$K' = 100$	$K' = 200$	$K' = 300$	$K' = 400$	$K' = 500$	$K' = 600$	$K' = 700$	$K' = 800$	$K' = 900$	$K' = 1000$
(α_l, β_l)	1	1	1	1	1	1	1	1	1	1
(α_l, β_m)	1	1	1	1	1	1	1	1	1	1
(α_l, β_h)	1	1	1	1	1	1	1	1	1	1
(α_m, β_l)	0.79	0.76	0.67	0.88	0.89	0.92	0.93	0.94	0.91	0.89
(α_m, β_h)	0.2	0.02	0.08	0.05	0.06	0.13	0.1	0.18	0.35	0.29
(α_h, β_l)	1	1	1	1	1	1	1	1	1	1
(α_h, β_m)	1	1	1	1	1	1	1	1	1	1
(α_h, β_h)	1	1	1	1	1	1	1	1	1	1

REPRODUCED:

$K' = 100$	$K' = 200$	$K' = 300$	$K' = 400$	$K' = 500$	$K' = 600$	$K' = 700$	$K' = 800$	$K' = 900$	$K' = 1000$
1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1
0.82	0.72	0.77	0.72	0.68	0.66	0.56	0.58	0.5	0.44
0.05	0.01	0.01	0.04	0.06	0.12	0.14	0.21	0.27	0.21
1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1

4.21. TABLE OA.3: STABLE CHOICES OF HYPERPARAMETERS.

ORIGINAL:

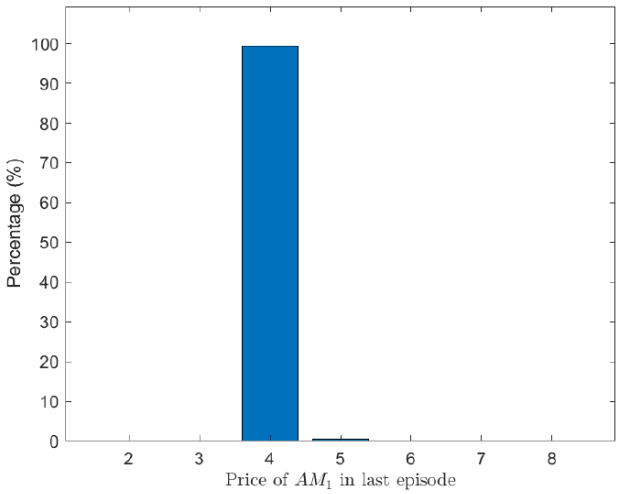
	(α_l, β_l)	(α_l, β_m)	(α_l, β_h)	(α_m, β_l)	(α_m, β_m)	(α_m, β_h)	(α_h, β_l)	(α_h, β_m)	(α_h, β_h)
(α_l, β_l)	0	0	0	0	0	0	0	0	0
(α_l, β_m)	0	0	0	0	0	0	0	0	0
(α_l, β_h)	0	0	0	0	0	0	0	0	0
(α_m, β_l)	0	0	0	1	0	0	0	0	0
(α_m, β_m)	0	0	0	0	0.29	0.4	0	0	0
(α_m, β_h)	0	0	0	0	0.4	0.59	0	0	0
(α_h, β_l)	0	0	0	0	0	0	0	0	0
(α_h, β_m)	0	0	0	0	0	0	0	0	0
(α_h, β_h)	0	0	0	0	0	0	0	0	0

REPRODUCED:

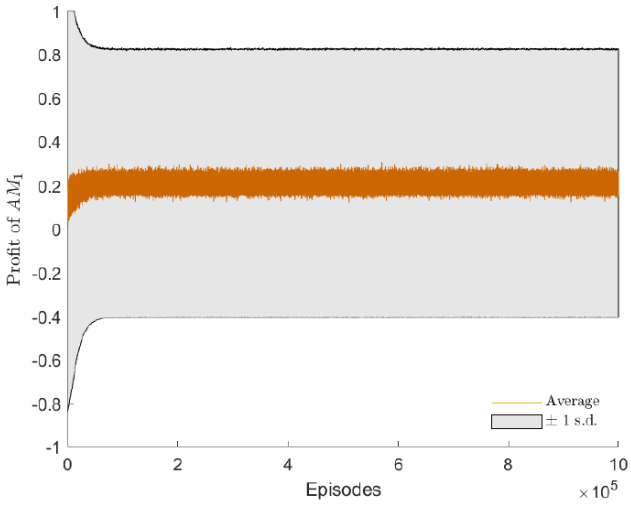
(α_l, β_l)	(α_l, β_m)	(α_l, β_h)	(α_m, β_l)	(α_m, β_m)	(α_m, β_h)	(α_h, β_l)	(α_h, β_m)	(α_h, β_h)
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	0	0	0.21	0.28	0	0	0
0	0	0	0	0.28	0.72	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

4.22. FIGURE OA.6: TWO AMS. DISTRIBUTION OF THE PRICE CHOSEN BY EACH AM IN THE LAST EPISODE, EVOLUTION OF AVERAGE PAYOFFS OVER EPISODES.

ORIGINAL:

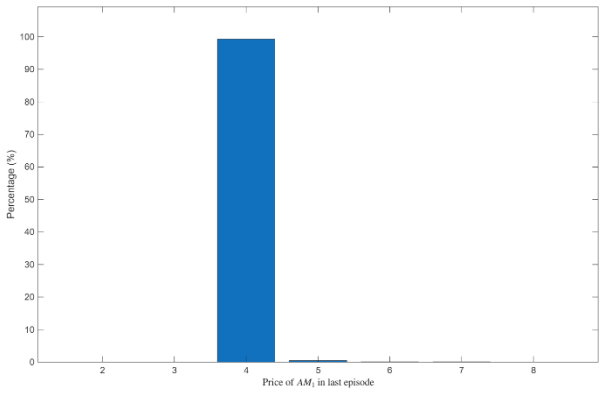


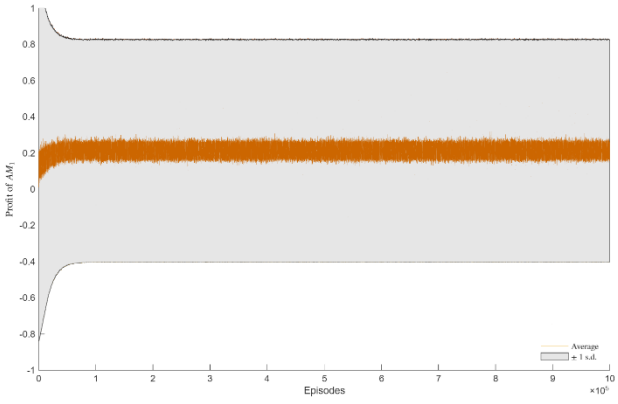
(a) Distribution of AMs' quotes. Almost all quotes are at 4, and the rest at 5.



(b) Evolution of AMs' average profits: Average profits reach around 0.2 after 20,000 episodes and then remain at that level, with a standard deviation around 0.6.

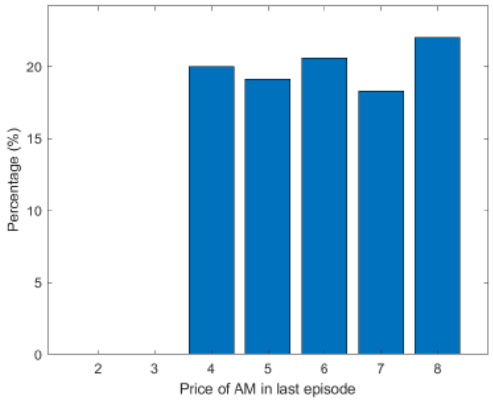
REPRODUCED:



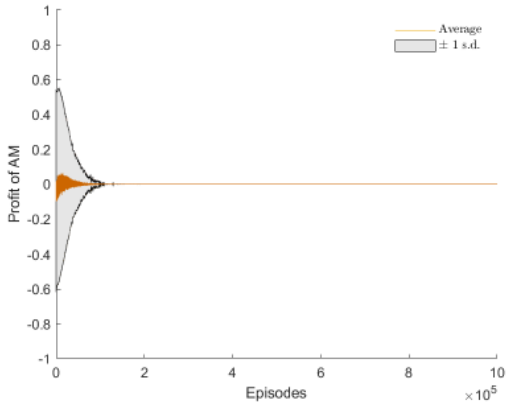


4.23. FIGURE OA.7: THE OM PLAYS "EXCLUSION". DISTRIBUTION OF THE PRICE CHOSEN BY EACH PLAYER IN THE LAST EPISODE, EVOLUTION OF AVERAGE PAYOFFS OVER EPISODES.

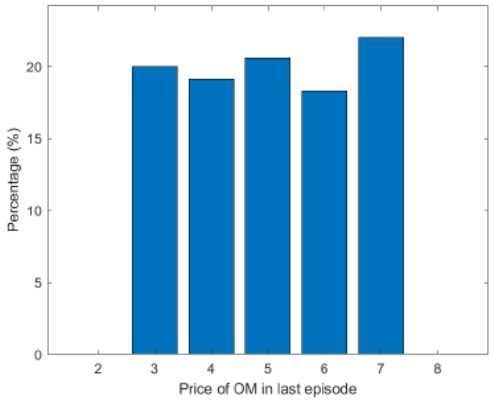
ORIGINAL:



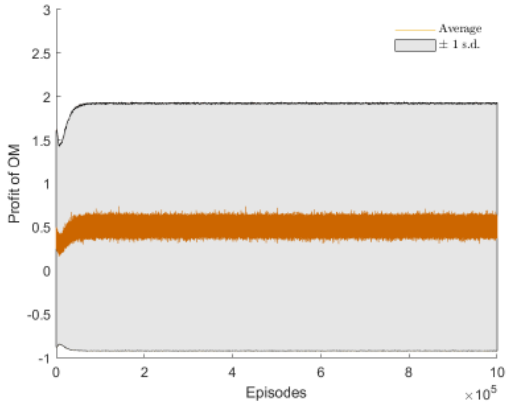
(a) Distribution of the AM's quotes: Quotes are approximately uniformly distributed between 4 and 8.



(b) Evolution of the AM's average profits: Average profits converge to around 0 after 100,000 episodes, with a standard deviation close to 0.

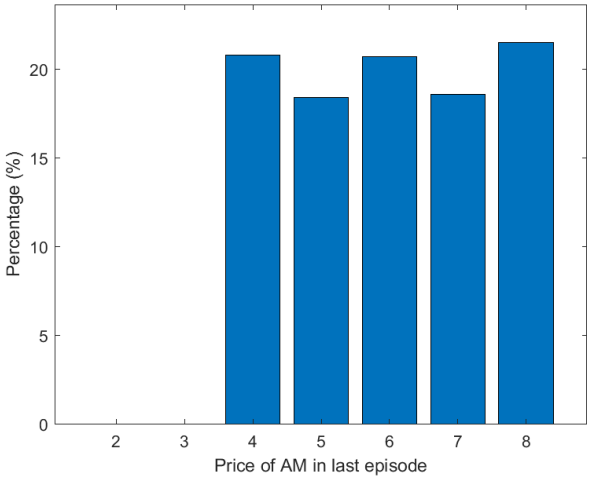


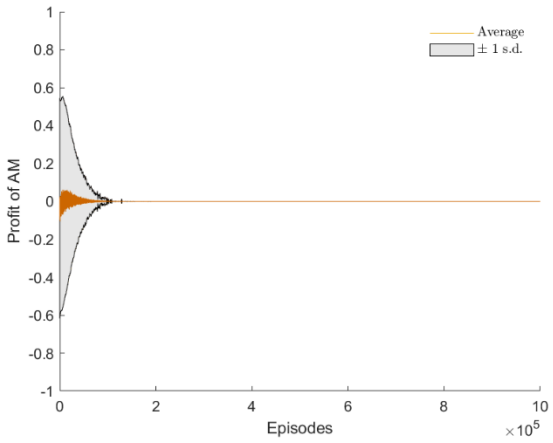
(c) Distribution of the OM's quotes: Quotes are approximately uniformly distributed between 3 and 7.

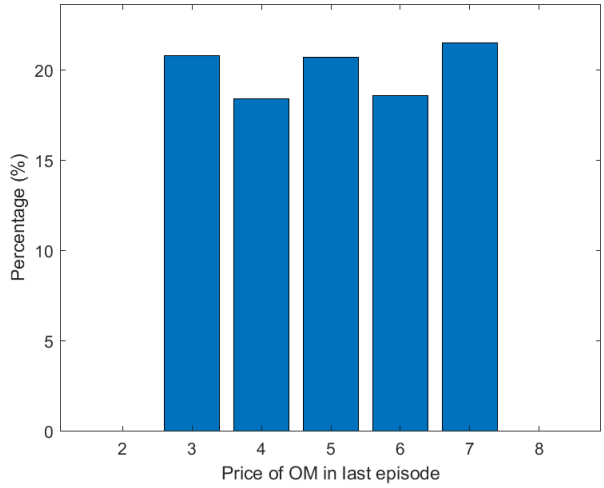


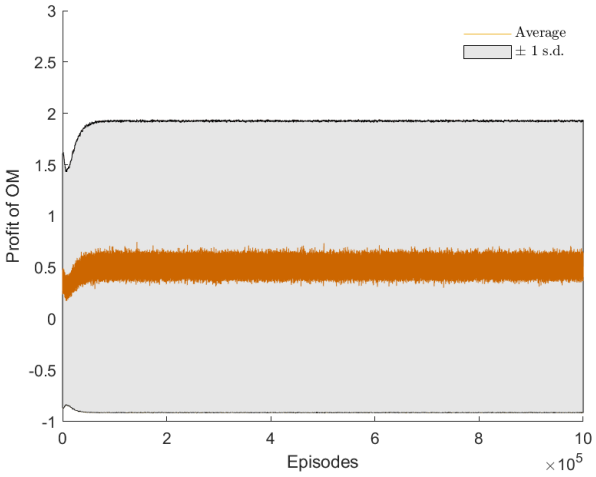
(d) Evolution of the OM's average profits: Average profits converge to around 0.5 after 100,000 episodes, with a standard deviation around 1.5.

REPRODUCED:





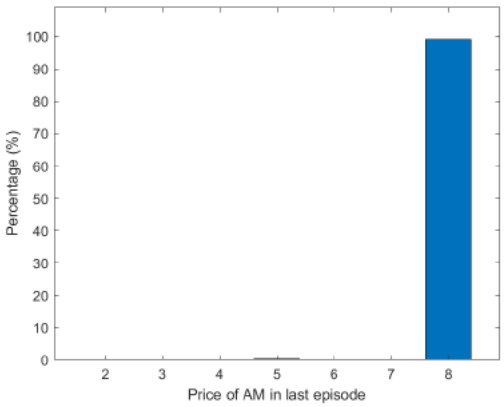




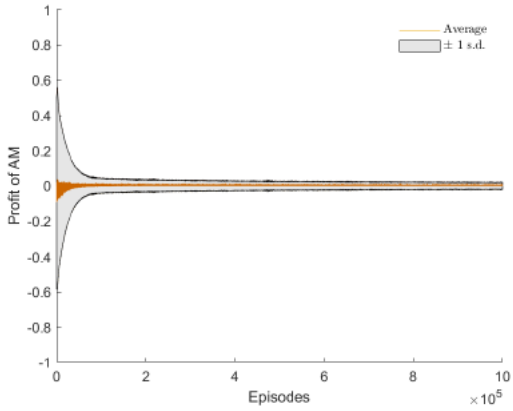
The distributions in Panels (a) and (c) have similar shapes but the values are slightly different. For instance, the leftmost bars in both original panels reach the 20 % threshold, whereas it goes above in ours.
Panels (b) and (d) are identical.

4.24. FIGURE OA.8: THE OM PLAYS "ACCOMMODATION".

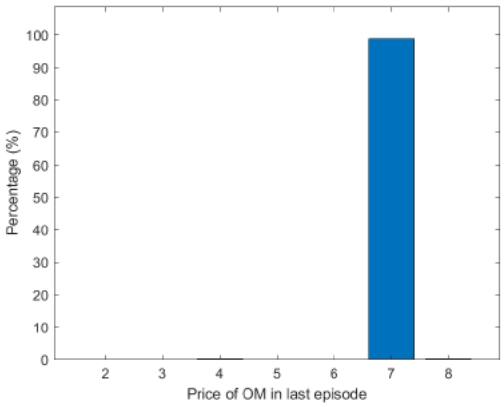
ORIGINAL:



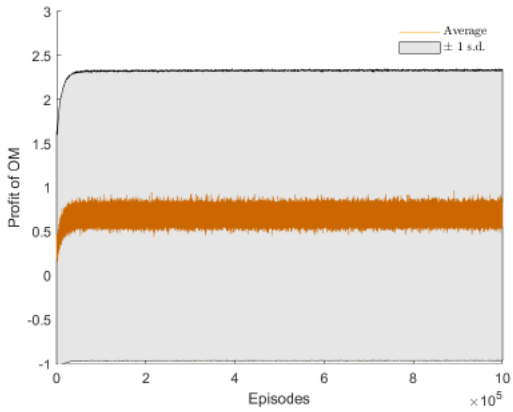
(a) Distribution of the AM's quotes: Almost all quotes are at 8, and the rest at 5.



(b) Evolution of the AM's average profits: Average profits converge to around 0 after 100,000 episodes, with a small but positive standard deviation.

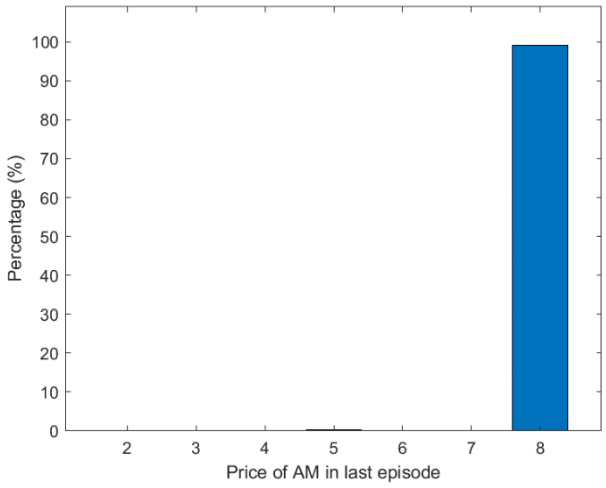


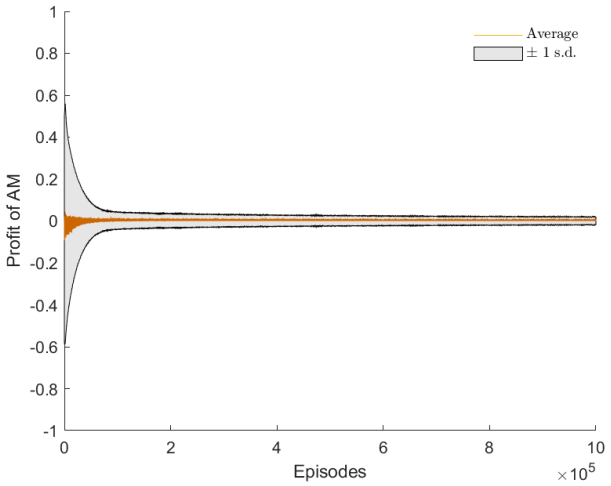
(c) Distribution of the OM's quotes: Almost all quotes are at 7, and the rest at 4.

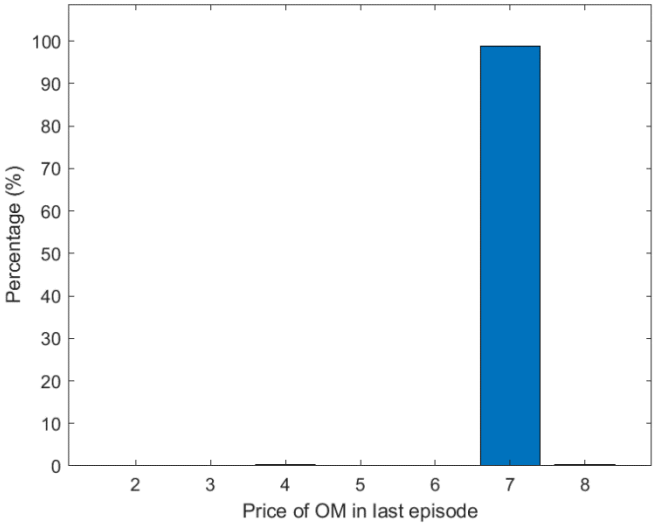


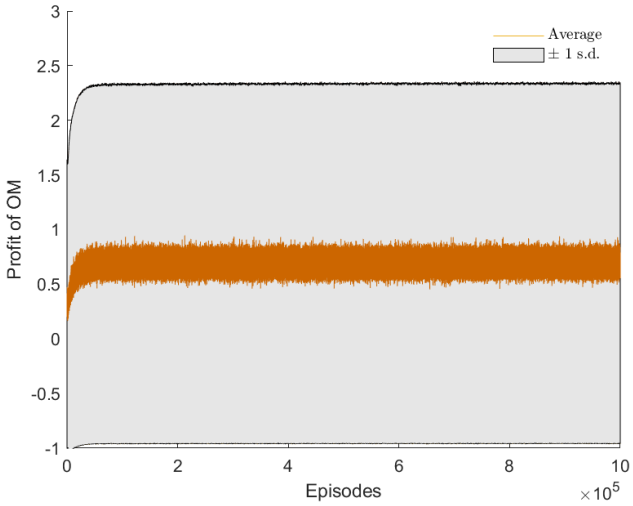
(d) Evolution of the OM's average profits: Average profits converge to around 0.6 after 100,000 episodes, with a standard deviation around 1.5.

REPRODUCED:


Percentage (%)
Price of AM in last episode


Average
± 1 s.d.
Profit of AM
Episodes
×10^5


Percentage (%)
Price of OM in last episode


Average
± 1 s.d.
Profit of OM
Episodes
×10^5